PHILOSOPHY SOUTH

What the Tortoise will say to Achilles – or "taking the traditional interpretation of the sea battle argument seriously"

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ABSTRACT

This dialogue between Achilles and the Tortoise – in the spirit of those of Carroll and Hofstadter – argues against the idea, identified with the "traditional" interpretation of Aristotle's "sea battle argument", that future contingents are an exception to the Principle of Bivalence. It presents examples of correct everyday predictions, without which one would not be able to decide and to act; however, doing this is incompatible with the belief that the content of these predictions lacks a truth-value. The cost of using a non-classical logic to cope with that may be too high for Stagirite's defenders, and they would still need to explain why our ordinary predictions seem to have a binary truth-value. In the end, the paper suggests that the problem of future contingents – and of free will – is not a logical problem at all, but rather a limit on what an agent can believe before taking a decision.

Keywords: future contingents, bivalence, beliefs.

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Achilles and Tortoise are talking, while playing *Battleship*.

Achilles: ...then Aristotle concludes that infinity, in this case, only exists *in potentia*. So as I knew, I will overtake you. QED.

Tortoise: You are certainly confident. But notice that is a *non sequitur*: you have just proven you *can* reach me, but you don't know you *will* reach me, necessarily. Unless you have been converted by Zeno to determinism. B5?

A.: Water! Well, proving it would be logically impossible. After all, as a *gaucho* teacher shall prove in some years (Barbosa Filho, 1999, p. 15), acting is to *make* a proposition come true, to make certain what is not. Therefore, we cannot know future contingents – on which we are still about to act. B4?

T.: Water! Could you explain it a little better? A6.

A.: As Aristotle (1963, p. 50, *De Int 9*) will say in a few years, suppose we predict there will be no sea battle tomorrow; suppose that's the case – it shall not happen. This implies it was true, yesterday, that it would not take place, and so the day before that. But if that proposition was *always* true, it means the naval battle could never occur; it is a logical necessity and, unless we return to our previous unpleasant discussion about *modus ponens* (Carroll, 1895), logic is inescapable. So if our forecasting was right, this battle would be impossible; if wrong, the battle would be

necessary. But that is absurd – it means we wouldn't have any kind of free will (because everything that will occur would be bound to occur), and life would be meaningless: "a tale told by an idiot, signifying nothing".

T.: You know, you really remind me the story of Arjuna, an Indian warlord who, in front of the desolating perspective of the war to come, considered becoming a pacifist on the eve of the great battle; his advisor Krishna (actually, a god's *avatar*) rebuked him, saying everything was already determined, and that, as a matter of fact, those warriors who would die were already dead; so the best Arjuna could do would be getting into the fight to attain glory. But Arjuna still wondered: if it's all previously determined, then why fight (Prasad, 1996, *Bhagavad-Gita*, III: 1-2)?

A.: That's my point. So Aristotle's solution (at least according to the so-called traditional interpretation) is to abandon the Principle of Bivalence: although it is neither true that tomorrow there will be a sea battle, nor that there won't be, the disjunction is true: "either there will be a battle, or there won't." This preserves our free will (Schmidt, 2009, p. 8). After all, if the future is still underdetermined, there is no fact that corresponds to such sentences to make them true; how could we attribute a truth-value to them?

T.: I am not fully convinced that this interpretation is not a subtle version of the modal fallacy, as G.E.M. Anscombe would say (*in* Sorensen, 2003, p. 122-123); after all, what does this use of "certain", "determined" or "necessary" add to the notion of a true proposition? Furthermore, with the principle of bivalence, we apparently also sacrifice the idea that a disjunction is a truth-function of the disjoints. This can't be done without a price, since it is from this truth-functional relationship that we can, among other things, make certain but inaccurate forecasts, such as "one day I will die" – equivalent to "I'll die tomorrow, or the day after that, or the day after that, etc.".

A.: You may be right. Two objections, however: as a criticism to the Stagirite, it would be anachronistic, since we are not dealing with anything like a Tarskian theory of truth (Barbosa Filho, 2004). If you want to do this, you should pick another target, such as Łukasiewicz's (1970, p. 153-178) many-valued logics. Or even someone who, despite being chronologically posterior to us, is *Prior* in name – he will develop a beautiful theory to deal with the truth-functionalist criticism (Prior, 1967).

T.: Sure, it would be fun to use non-Aristotelian logics[2] to save Aristotle. And I do love the idea of infinitely many truth-values, we could argue about them forever…

A.: Please, don't. I take back what I have just said.

T.: … but I'm no expert on modal logic, more so if it demands dealing with Polish notation; sometimes I need to go slowly even with something as simple as *modus ponens*, as we have already noted on another occasion. My point is much less technical than that; I'm not even sure if it's a general argument in favor of bivalence, as we shall see – or if it applies only to those who reject it in order to maintain a correspondence theory of truth.

If you will allow me, let's move on to a new path, so we may *converge*: you said you would reach me and committed yourself to this proposition's truth. Now you might feel tempted to say you were wrong; this would be a normal error, as anyone can make when trying to make ordinary predictions. Nevertheless, we cannot refrain from making them – some of them wrong, some of them right: it would be impossible, e.g., to plan and to rule (to take decisions on interest rates, agriculture, war, etc.) if we could not believe that things are going to be a certain way – although this belief may be false or inaccurate. All of our actions, in particular those relating to others, aim at a future state of things that we want to produce. And we could not aim at anything if we could not think "if I do this, he will do that. So, I will do x; he will do y". Otherwise, it would result in the widespread unemployment of strategists; "that would be complete anarchy in the conduct of men, with experience being plainly in contradiction. Such an assumption would make the existence of economy just as impossible as that of economic theory" (Morgenstern, 1976, p. 175).

So we could add: if knowledge of a proposition is inconsistent with acting on it (since it is the action that turns it into a true proposition), it is also impossible to act without believing that several future contingents are true. Call it the *doxastic* requirement: how can we abdicate from the knowledge (and truth) of *p*, and still *believe* in *p*[3]?

Worse, there is a range of decisions (if I can call them that) which sentient animals such as us do not even need to make consciously, because we know that a state of things will occur, although it is a future contingent subject to one's decision. I know, at this very moment, you will not get naked,

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nor commit suicide, nor leave without saying goodbye. I know this because I know you are a rational animal, with certain interests and instincts; I know you have many reasons for not doing so, but no reason to do so (save, perhaps, to contradict me). I know it, even if the hypothesis has never crossed your mind. And I can apply the same kind of forecasting to any normal intelligent animal – with the possible exception of some existentialist philosophers.

However, if the traditional interpretation were the correct theory of future contingents, even if you did decide to take your clothes off, we should not believe that it would be true that you would do it – you could still change your mind until the last moment. So, even your decision could not fill this truth-value gap until the action took place.

A.: That is curious, as Aristotle apparently does not ignore the relationship between beliefs and future states. After all, if every action aims at some good, at its end (Aristotle, 2009, p. 3, EN, 1094a1), then the agent must believe that the good must follow from the action; i.e., beliefs and desires necessitate action (Aristotle, 2009, p. 119, EN, 1145b25-31). He does not seem to see any incompatibility between the theory of action and moral responsibility and the notion of psychological determinism related to the formation of character (Aristotle, 2009, p. 47, EN, 1114a16-23) rooted in our Greek culture – the idea that our actions are determined by who we are, by the way character is built. But if psychological determinism is not inconsistent with the notion of moral responsibility (Dworkin, 2011, p. 236-7), why should the "necessity of truth" (to use an expression from Barbosa Filho, 2004, p. 234) be so? As Hardie (1968, p. 278) might ask, which of the two versions of Aristotle is the real one: the libertarian or the compatibilist – even if only a "psychological compatibilist"?

T.: Both, perhaps. Maybe he will never be fully convinced of any of these theories. Alternatively, he may write a great dialogue on the subject – and it may be lost forever. Some interpreters may disagree with me, but I believe we will never know it; even so, I do not think there is no truth on this matter.

A.: We could say, however, that beliefs about future contingents embed probability statements; and it would seem reasonable to assess their truth-values as real numbers between [0, 1], applying the corresponding principles of the probability calculus (Łukasiewicz, 1970, p. 47-48). This would cope with your requirement that a theory of truth must be compatible with belief in future contingents.

T.: Indeed, it would. First, notice that it would not vindicate the "traditional interpretation", since this derives from a common sense view of correspondence, the thesis that future contingents have no truth-value – instead of lots of them, as you propose. As I said before, you are trying to rescue the Stagirite with non-Aristotelian logics.

Besides, you would still owe me an explanation of why I think:

(i) my beliefs about the future are attitudes committed to the truth of their contents, which refer to future events;

(ii) a proposition P that refers to a fact F is the same if it is said before (i.e., as a prediction) or after the occurrence of F (i.e., as a description);[4]

(iii) there is a difference between saying "there will be a sea battle" and "it is likely there will be a sea battle" (which explains (i));

(iv) this second sentence ("it is likely…") justifies the belief in the first ("there will be…"), and the truth of the second is usually explained by appealing to the truth of the first – I would even say the relationship follows something function-like, perhaps in the manner of what Sorensen (2006, p. 607) argues: if a sentence p lacks a truth-value, the modest sentence "perhaps p" also lacks it. If, to give a crude example, there are nine times more possible worlds in which P occurs (or if the frequency of this event is nine times greater than its opposite), we can say that it is likely that P.

It would be curious to be wrong on the first point, about the content of my belief; moreover, I wonder what it would be like to believe that I am wrong in this case – not in the sense that the content of my first-order beliefs are false, but "wrong" in the sense that, in fact, I do not know what these beliefs are! And this happens not only in exceptional cases, such as when someone professes an absurd belief, but also in a routine and systematic way.

A.: Yet, you do believe it is logically possible to believe the impossible;[5] it's (apparently) logically possible that you *are* wrong, and that propositions may have a *continuum* of truth-values – that, e.g., "there will be no sea battle tomorrow" is verified by 95% of the values of the variables (in this case, possible states of affairs for tomorrow). Actually, there are reasons to believe that our basic inferential procedures are Bayesian (Oaksford and Chater, 2009); and statisticians and risk managers do talk in this way: "there's only a 5% chance that our loss will exceed our Value-at-Risk", or "with a confidence level of 95%, our result will be inside our margin for error of 2%". Perhaps you should do the same.

T.: But I do not; I speak ordinary English, like everyone else. Besides, we could still ask these people if their forecasts

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[4] Perhaps it does not entail that both *statements* have the same truth-value. For MacFarlane (2003), e.g., the truth-values of these depend on the context of assessment – even though he thinks they are the same proposition (but with different truth-values).

[5] Sorensen (2001, p. 124-125) presents a "transcendental argument" (supposedly) proving we can believe inconsistencies – but it does imply that we do not *know* these beliefs to be inconsistent. However, such "impossible" beliefs are exceptional (although inescapable: reason demands we have them), such as beliefs about vagueness ("there's no last 'noonish' minute"). Could we say the same about future contingents?

are *true*, if they *believe* them. After all, if they said "yes", bivalence returns through the back door, since "it is likely there'll be a battle tomorrow" is either true *now*, or false *now* – although the value of "there will be a battle…" may change as time passes and its *posterior* probability is updated. If this is what I mean by "there will be a sea battle tomorrow", then we can conclude that what I say has a well-defined truth-value *now*.

A.: Ok, but your meta-truth-value "truth now" may be in a *continuum*, too. One could say "there's a 99% chance that there's a 95% chance that…". Meta-analysts do this all the time.

T.: No problem; I'll be satisfied with the meta-meta-statement "it's true that it's likely that it's likely that…". If we do get to the end of this, it is the notion of contingency that may vanish, not truth.

A.: Wait… this is fun, but, if we proceed, we will go on forever – *again*. We have agreed to avoid that. So, let me restate your previous point, about usual predictions; you said that we know that we will not be naked, nor commit suicide…

T.: No! I said *I* knew *you* would not be naked, nor commit suicide. I said nothing about you knowing anything about me.

A.: I do not understand. Sure, if you think that about me, I can think the same about you, too. And you said "any normal human being"… And you would have no reason to do any of that, to kill yourself, or…

T.: Or getting naked? Of course. First, I am not a human being, nor do I consider myself normal – from my point of view, I am a very special reptile. And I am already naked: I cannot imagine what a chelonian in clothes would be like – something as unthinkable as a talking lion. As for suicide… well, a philosopher with a talent for writing will say, one day, that this is the real philosophical question. Moreover, leaving aside philosophers and terminally ill people, a person's suicide is often a surprise – after all, if it were publicly planned, the victim's friends would hardly have allowed it. It would also be a way to prove that you cannot reach me, and a dramatic outcome for this dialogue.

A.: But you are not really thinking about it. I know you will not do it; especially because it would be immoral, and you are a virtuous tortoise.

T.: You may be right; but if modesty is a virtue, I cannot say that I am virtuous either (Sorensen, 1988, p. 9). My point is that I could not believe it, until the moment of the decision; and after having made the decision, I could not avoid believing it, without reviewing the very same decision. Believing in "I will do x, but didn't decide it yet" would be similar to believing in the statement "it rains, but I do not believe that it rains." Let's say I have a perspective of myself that, although privileged, has some "blind spots" (Sorensen, 1988). But I do not claim that my *blind spots* apply to you; you can believe whatever you *decide* to believe.

A.: This reminds me of the time we met in Paris (Hofstadter and Dennett, 1981, p. 430), when you made me imagine what it would be like to read a book containing a complete description of my brain in order to predict how I would act. And suddenly I found myself consulting the book *ad infinitum,* since I had to check the book to update my knowledge about my brain-states after having checked the book.

If I have understood you correctly, your main point is not properly logical, but doxastic, and it is less about action theory than about decision theory. It relates not to the study of the truth-value of predictions, but to what we need to believe in order to act (our forecasts) and to what we cannot believe (that is, the content of the decision itself, before it occurs). The agent must be able to believe that others will act in a certain way (although it is possible that they do not act that way), but she cannot believe that she will act in a certain way without deciding so. Of course, she may feel tempted to collapse this *blind spot* into logical indeterminacy (as does McKay, 1960).

Which leads to a question: what of when another's action is related to my decision? I try to predict what the other will decide, and I will make my decision on that basis; but he will be doing the same. As when I was wondering where on the board you'd put your ships – knowing that you knew I was trying to guess their position… in fact, sorry, what was your last move?

T.: A6. That is why someone will invent game theory (Morgenstern and Neumann, 1953). Usually, it does not imply anything paradoxical: assuming you are rational, as am I, we know it would not be worth devoting my time and energy to find the set of optimal strategies for ships' distribution, which is why we distribute them randomly; in fact, it is likely that this *is* the optimal distribution strategy (a mixed-strategy, if you like) – as long as we avoid situations in which the ships get clustered. I assume you did it as well – because if I thought otherwise, it would be better for me to review my strategy. Do I know that? It depends, in part, on whether it is a safe or sufficient justification for my conclusion.

I think that's the real problem behind the sea battle debate: imagine that, in a possible world (let's call it "Greece-1"), two generals (call them Mr. T and Mr. A) are wondering if they are going to engage in a fight on the next day, and both of them ignore that the battle will be impossible because of a disaster, like a tsunami; there is no *ontological* indeterminacy in this case[6] (one cannot fight a battle while the earth is falling apart), only epistemic and *doxastic* indeterminacy – i.e., the generals don't know there will be no battle. It suggests that any alleged indeterminacy, in this scenario, comes from the decision-makers, not from the environment.

In another possible world ("Greece-2") accessible from that, but disaster-free, Mr. A thinks "No way! There will be no battle, I am sure, since I am defecting to Mr. T's side". But in that world, there's no more indeterminacy, too, at least for

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[6] If you think natural disasters are ontologically undetermined until they are predicted, imagine that some super scientist or prophet did see it coming.

A; he cannot but *believe* there will be no sea battle, since he's decided to avoid it (and it takes two to tango). Funny thing, he could neither believe in his defection, until he decided to; actually, he can still change his mind (which means that it is still *possible* that the battle happens), but he cannot believe he will do it, as long as he's decided to defect.

However, until T is aware of this fact, he may still believe in the indeterminacy of the battle. And that is the real issue: both generals deemed the battle indeterminate because neither knew his own decision (because neither of them had made it yet), nor that of his opponent (because, again, none of them had made it yet – usually, one needs to know what he's responding to in order to give the best response). Imagine ("Greece-3") that both generals have common knowledge of their commitment to the battle, or even ("Greece-4") to the peace; then there would no longer be any doubt.

A.: It is hard to imagine how, starting from common ignorance, this common knowledge could be achieved in a finite and discrete series of steps; Gen. T sends a signal *1* that he wants to fight, then Gen. A receives it and answers with a signal *2* that he wants to fight too, which is, later, received by T – who responds with signal *3* that he's received *2* and is still going to fight, etc. This may seem implausible at first glance – but then it reminds me all the times I have had trouble in scheduling lunch with friends or setting a date; I think that is what they will call the "Byzantine Generals Problem" and the "E-mail Game" (Binmore, 2007, p. 170-171). It is worse than trying to race with you.

T.: Right, and despite it, we usually meet at the right time and place – and it is going to get easier when some technology offers us the possibility of sending and receiving reliable signals simultaneously. We could not meet if there was real doubt – if none of us believed the meeting was going to take place; we can change our minds, of course, but then we will change our beliefs also. And we may be mistaken; but that's ordinary ignorance, and we don't need indeterminacy to explain it.

The ontological puzzle is: how could ontological indeterminacy be affected by one-sided decision-making? How could it be affected, actually, solely by any-sided decision-making? The battle is still possible, but A cannot see it as indeterminate; and it's determinate for T, in Greece-2, only because of his ignorance.

Even if there is an explanation for such ontological indeterminacy, it lacks the appeal and practical relevance it seemed to have when we first considered the sea battle argument; now, what really should be explained is the relationship between an agent's beliefs (including his beliefs about himself) and his own decisions.

A.: … A6, right? Water. This discussion has reminded me of the son of an acquaintance: to revenge his father's death, he killed his own mother because the Oracle so foretold (Aeschylus, 1986). I do not remember when it happened. But at the time of the final blow, the boy hesitates and asks an accomplice: "shall I kill my mother?" Then his friend reminds him of his obligation to Apollo and his oath.

He did not fail to believe that what the Oracle says is true – yet, he had doubts in the last moment. Otherwise, how could we call this a decision (Williams, 1993, p. 138)? This vindicates the main objective of Barbosa Filho (2004): the incompatibility between knowing and doing – at least before the decision is made, and as long as it is possible. E3.

T.: I can empathize with that boy. Even if I were just a character in a story, I could not seriously believe it possible for me to decide to *p* without believing that I will *p*, and (usually) vice versa; nor could even the author of the story make me believe it. But in this case, one could say that I would not believe anything; there would be no beliefs, because there would be no Tortoise to do the believing. However, if that were the case, I could not believe it either: it would be like believing in the sentence "I do not exist" – which is no logical absurdity, since its token will be true, for each and every mortal.

Alas, water!

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