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A dialogical frame for fictions as hypothetical objects[1]

Um quadro dialógico para ficções como objetos hipotéticos

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Abstract
Recent work on the development of a dialogical approach to the logic of fiction stresses the notion of *existence as choice*. Moreover, this approach to existence has been combined with the notion of ontological dependence as deployed by A. Thomasson's artifactual theory of fiction. In order to implement such a combination within the dialogical frame several predicates of *ontological dependence* have been defined. However, the definition of such predicates seems to lean on a model-theoretic semantics for modal logic after all. The main aim of the present paper is to set a dialogical frame for the study of fictions in the context of the dialogical approach of CTT recently developed by S. Rahman and N. Clerbout where a fully-interpreted language is unfolded. We will herewith develop the idea that in such a setting fictional entities are understood as hypothetical objects, that is, objects (functions) the existence of which is dependent upon one or more hypotheses that restrict the scope of choices available. We will finish the paper by suggesting that this provides both a natural and genuinely dialogical way to understand R. Frigg's take on scientific models as fictions and a new perspective on Thomasson's notion of *generic ontological dependence*.

Keywords: fiction, dialogues, dialogical logic, hypothetical objects, scientific models.

Resumo
Trabalhos recentes sobre o desenvolvimento de uma abordagem dialógica para a lógica da ficção enfatizam a noção de existência como escolha. Além disso,

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esta abordagem da existência tem-se combinado com a noção de dependência ontológica implantado pela teoria artefactual da ficção de A. Thomasson. A fim de implementar essa combinação dentro do quadro dialógico, foram definidos vários predicados de dependência ontológica. No entanto, a definição de tais predicados parece basear-se em uma semântica do modelo teórico para a lógica modal, afinal. O objetivo do presente trabalho é estabelecer um quadro dialógico para o estudo de ficções no contexto da abordagem dialógica dos CTT recentemente desenvolvida por S. Rahman e N. Clerbout, onde uma linguagem totalmente interpretada é desdobrada. Vamos, com isto, desenvolver a ideia de que, em tal cenário, as entidades ficcionais são entendidas como objetos hipotéticos, ou seja, objetos (funções) cuja existência é dependente de uma ou mais hipóteses que restringem o escopo de opções disponíveis. Vamos terminar sugerindo que isto fornece uma maneira natural e genuinamente dialógica para compreender modelos científicos como ficções (Roman Frigg) e uma nova perspectiva sobre a noção de dependência ontológica genérica de Thomasson.

Palavras-chave: ficção, diálogos, lógica dialógica, objetos hipotéticos, modelos científicos.

Introduction

A brief examination of the most recent literature in logic will make it apparent that a host of research in this area is devoted to the study of the interface between games, logic and epistemology. These studies provide the basis of ongoing enquiries into the history and philosophy of logic, going from the Indian, the Greek, the Arabic, the *Obligationes* of the Middle Ages to the most contemporary developments in the fields of theoretical computer science, computational linguistics, artificial intelligence, social sciences and legal reasoning. In fact, a dynamic turn, as J. v. Benthem puts it, is taking place where the epistemic aspects of inference are linked with game theoretical approaches to meaning[4]. In regard to the birth of this turn, it could be placed around the 1960s, when P. Lorenzen and K. Lorenz developed dialogical logic – inspired by Wittgenstein's language games and mathematical game theory – and when some time later Hintikka (1962, 1973) combined game-theoretical semantics with epistemic (modal) logic[5]. However, while Hintikka's approach is based on a model theoretic semantics, the dialogical framework is closer to the philosophical tenets underlying constructivism. Indeed, one possible way to link dialogues and constructivism is to follow M. Marion's[6] proposal to make use of Brandon's (1994, 2000) pragmatist take on inferentialism. Indeed, Brandon's pragmatist inferentialism is led by two main insights of Kantian origin (combined with pragmatism) and one that stems from Brandon's reading of Hegel, namely (i)

[4] New results in linear logic by Girard at the interfaces between mathematical game theory and proof theory on the one hand and between argumentation theory and logic on the other resulted in the work of, among others, Abramsky, van Benthem, Blass, van Ditmarsch, Gabbay, Hyland, Hodges, Japaridze, Krabbe, Prakken, Sandu, Walton and Woods. They all explore the scope of a new concept of logic in which logic is understood as a dynamic instrument of inference.
[5] If we were to pinpoint a precise date, the year 1958, the date of publication of ***Logik und Agon*** by Lorenzen, could be taken to mark the very beginnings of the dynamic turn.
[6] In fact, Marion (2006, 2009) was the first to propose a link between Brandom's pragmatist inferentialism and dialogical logic in the context of Hodges' (2001) challenges to the game theoretical approaches. Another relevant antecedent of the present work is the PhD-thesis of Keiff (2007), who provided a thorough formulation of dialogical logic within the framework of speech-act theory.

judgments are the fundamental units of knowledge, (ii) human cognition and action are characterized by certain sorts of normative assessment[7] deployed by games of giving and asking for reasons, (iii) communication is mainly conceived as cooperation in a joint social activity rather than as sharing contents.[8]

The crucial point of the epistemic approach is that assertion or judgment amounts to a knowledge claim. So, if meaning of an expression is deployed from its role in assertions (the linguistic expressions of judgments), then an epistemic approach to meaning results. In relation to the second point, Brandon implements the normative aspect with the help of Sellar's (1954) notion of games of *giving and asking for reasons*. Indeed, in Brandon's view, it is the chain of commitments and entitlements in a game of giving and asking for reasons that tights up judgment and inference.[9] Sundholm (2012) provides the following formulation of the notion of inference in a communicative context that can be also seen as describing the core of Brandon's pragmatist inferentialism[10]:

> *When I say* "Therefore" *I give others my authority for asserting the conclusion, given theirs for asserting the premisses.*[11]

Recent work on dialogical logic develops plays between commitments and entitlements in the context of a pragmatics theory of meaning for fictions with the help of the notion of *existence as choice*. In a nutshell: the use of a singular term is said to have ontological commitment iff it has been chosen while substituting a bounded variable that occurs in an existentially quantified expression. Accordingly, in an argumentative context, a proponent is not entitled to the thesis that, say, *Vampires exist*, by the opponent's concession that *Nosferatu* is such a creature –unless the opponent has also chosen *Nosferatu* to defend some existential claim: it is only the existential choice-commitments of the opponent that entitle the proponent to

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[7] The normative aspect, rooted on the shift from Cartesian ***certainty*** to ***bindingness of rules***, distinguishes Brandom's pragmatism of others: "One of the strategies that guided this work is a commitment to the fruitfulness of shifting theoretical attention from the Cartesian concern with the grip we have on concepts – for Descartes, in the particular form of the centrality of the notion of ***certainty*** […] – to the Kantian concern with the grip concepts have on us, that is the notion of necessity as the bindingness of the rules (including inferential ones) that determine how it is correct to apply those concepts" (Brandom 1994, p. 636).

[8] In relation to the model of holistic communication envisaged, Brandom (1994, p. 479) writes: "Holism about inferential significances has different theoretical consequences depending on whether one thinks of communication in terms of ***sharing*** a relation to one and the same ***thing*** (grasping a common meaning) or in terms of ***cooperating*** in a ***joint activity*** […]".

[9] Moreover, according to Brandom, games of asking for reasons and giving them constitute the base of any linguistic practice: "Sentences are expressions whose unembedded utterance performs a speech act such as making a claim, asking a question, or giving a command. Without expressions of this category, there can be no speech acts of any kind, and hence no specifically linguistic practice" (Brandom, 2000, p. 125).

[10] These remarks have already been pointed out in Clerbout and Rahman (2015), who also notice that although Brandom's approach and the dialogical frame share some important tenets, they are different frames after all. More precisely, although the pragmatist approach to meaning of the dialogical framework shares with Brandom's pragmatist inferentialism the claim that the meaning of linguistic expressions is related to their role in games of questions and answers and also endorses Brandom's notion of justification of a judgment as involving the interaction of commitments and entitlements, dialogicians maintain that more fundamental lower-levels should be distinguished (see Appendix I). Those lower-level semantic levels include (i) the description of how to formulate a suitable question to a given posit and how to answer it, and (ii) the development of plays, constituted by several combinations of sequences of questions and answers brought forward as responses to the posit of a thesis. From the dialogical perspective, the level of judgments corresponds to the final stage of the chain of interactions just mentioned. More precisely, the justifications of judgments correspond to the level of winning strategies, which select those plays that turn out to be relevant for the drawing of inferences.

[11] Actually, Sundholm (2012) bases his formulation on Austin (1961 [1946]).

bring forward existential claims.[12] The general dialogical approach to fiction has been introduced by Rahman *et al.* (1990), who summarize their proposal in the following way:

> Being a pragmatic and not a referential approach to semantics, dialogical logic does not understand semantics as mapping names, propositions and relationships into the real world to obtain an abstract counterpart of it, but as dealing (handeln) with them in a particular way. This allows a very simple formulation of free logic the core of which can be expressed in a nutshell, namely: in an argumentation, it sometimes makes sense to restrict the introduction of singular terms in the context of quantification to a formal use of them. That is, the proponent is allowed to use a constant iff this constant has been explicitly conceded by the opponent (Rahman *et al.*, 1990, p. 357).

Since this initial paper further important developments have been published by Redmond (2010), Fontaine and Redmond (2011) and Fontaine (2013) that delve into both the dialogical structure of several free logics and the dynamics aspects proper to argumentative contexts (see Appendix II). Moreover, in the latter publications the dialogical approach to existence as choice has been combined with the notion of *ontological dependence* as deployed by Thomasson's (1999) *artifactual theory of fiction*. In order to implement such a combination several predicates of *ontological dependence* defined purely by dialogical terms have been attempted. However, the definition of such predicates seems to lean on a model-theoretic semantics for modal logic after all.[13] This hinges on a general problem of the standard dialogical approach to meaning where the dialogical semantics affects only logical constants. In pursuance of filling that gap in the dialogical theory of meaning new researches have linked dialogical logic with Per Martin-Löfs (1984) Constructive Type Theory (CTT), where a fully-interpreted language is unfolded (see Rahman and Clerbout, 2013, 2014; Clerbout and Rahman, 2015; Rahman *et al.*, 2015).

The main aim of the present paper is to set a dialogical frame for the study of fictions in the context of the dialogical approach of CTT where fictional entities are understood as hypothetical objects, that is, objects (functions) the existence of which is dependent upon one or more hypotheses that restrict the scope of choices available. Such a reading of fiction as play-objects for open assumptions as resulting from a fully interpreted language go along with Brandon's (1994, p. 636) view on the commitments attached to singular terms in games of giving and asking for reasons.

Let us point out that, as we will briefly discuss at the end of the paper, although the frame to be developed herewith aims at fictions of any sort, by its very nature it seems to be close to the most recent fictionalist approaches to scientific models such as the one of Frigg (2010) and Godfrey-Smith (2006, 2009). In this context, if – ontologically speaking[14] – models are understood as fictions, this can be expressed with the claim that models are devices to reason provided some hypotheses and that given two hypotheses both deploy possible ways to complete our lack of knowledge either by presenting two incompatible alternatives or by establishing a relation between them such that the second hypothesis specifies the first one by adding information. The hypothetical objects are in fact variables of a certain type that require a definition in order to be applied. Moreover, in a dialogical frame,

[12] The idea behind makes use of the dialogical theory of meaning of quantifiers, where its local semantics is given by both a choice and the propositional result of this choice (see Appendices I and II). Rahman ***et al.*** (1990) introduced a sequence of quantifiers.

[13] For a modal theoretic semantics of the notion of ontological dependence see Fontaine and Rahman (2013).

[14] In fact, when we say that models are fictions we are claiming that ontologically speaking fictions and models are on a par, not that they ***role*** or use is the same– see Giere (2009).

hypotheses (possible worlds) as specifications are the result of answers to questions posed in relation to a given hypothesis. We will finish the paper by suggesting that this frame provides a natural and genuinely dialogical way to understand Thomasson's notion of *generic ontological dependence.*

The Dialogical Frame

Worlds as hypotheses and dialogical free logic revisited

If we are going to deal with fictions and those are to be considered to be some kind of objects in a dialogical frame, what we need is to extend this frame in such a way that those objects are understood as some kind of intentional objects. In fact there is already some work on dialogical modal logic (such as Rahman and Rückert, 2001; Redmond and Fontaine, 2011; Clerbout, 2014) where intentional objects are formulated within a first-order modal frame. However, this approach that makes use of labels is too close to the model-theoretical approach of Kripke's modal logic: in such a setting, labels are names of model-theoretic worlds after all. In other words, what we need is a conception where worlds are introduced at the object-language level and have a genuine dialogical meaning rather than a metaphysical nature. Thus, since our aim is to formulate fictional objects as hypothetical entities within a dialogical framework, a dialogical CTT-approach to modal logic is due. Fortunately, Ranta (1991, 1994) provided one half of the task by developing a CTT version of possible world-semantics. What we need now is to combine it with the dialogical approach to CTT.

Let us start with the CTT approach to possible worlds as developed by Ranta (1994; and see also Primiero, 2008, p. 150-158). The main idea of Ranta is that an assertion relativized to a possible world W amounts to a hypothetical where that assertion is brought forward provided the hypothesis *W* and this is expressed at the level of the object-language. In other words, the modal assertions are reduced to hypotheticals. Thus, one might say that, in some way, Leibniz's (metaphysical) notion of possible world is placed into Kant's conception of hypotheticals. Thus, what we need is not labels for worlds but assertions that are brought forward during a play provided some open assumptions. Accordingly, hypotheses take the place of worlds and arbitrary elements (variables) of the hypotheses the proposition is dependent on take the place of world-labels. More generally, and independently of the dialogical setting, this yields the following correspondences – pointed out by Ranta (1991, p. 83):

A : set in W means $A(x)$: *set* $(x : W)$
$A = B$: I in W means $A(x) = B(x)$: *set* $(x : W)$
$a : A$ in W means $a(x) : A(x)$ $(x : W)$
$a = b : A$ in W means $a(x) = b(x) : A(x)$ $(x : W)$

The CTT take on hypotheses is the following: Asserting *A is true provided the hypothesis W* amounts to the assertion *There is a proof of A such that is dependent on some (yet unknown) proof of W*. We use a variable *x* for such a proof in the same way as we make use of a variable when considering an arbitrary element of a set. In the case of the dialogical approach to CTT, proof-objects only occur at the level of strategies, however, at the play level, play-objects provide the ontology suitable for that level. More precisely, while bare play-objects provide the ontology of categorical moves, functions (with variables as arguments) provide the play-objects of hypotheticals. As we will discuss below, hypothetical objects in general and fictions in particular are understood in this setting as *dependent* objects. Let us now make use of the translation proposed by Ranta in order to formulate in the context of dialogical logic the local meaning of quantifiers brought forward by hypothetical moves – and, in doing so, introduce its modal structure at the level of the object-language:

In order to avoid a heavy notation we will not write the world variable in proposition but we will assume that it occurs free in it; for instance the expression b(y) : (∃x : A)φ (y : W) should be read as b(y) : [(∃x : A)j](y) (y : W) (see Table 1).

Table 1. Local meaning of quantifiers with hypotheses.

Posit	Challenge	Defence
X $b(y) : (\exists x : A)\varphi\ (y : W)$	**Y** $?_{prop}$	**X** $(\exists x : A)\varphi : prop, A, W : set$
	Y $?_L$	**X** $L^{\exists}(b(y)) : A\ (y : W)$
	Or	Respectively
	Y $?_R$ **[the challenger has the choice]**	**X** $R^{\exists}(b(y)) : \varphi(L(b(y)))\ (y : W)$
X $b(y) : (\forall x : A)\varphi\ (y : W)$	**Y** $?_{prop}$	**X** $(\forall x : A)\varphi : prop, A, W : set$
	Y $L^{\forall}(b(y)) : A\ (y : W)$	**X** $R^{\forall}(b(y)) : \varphi(L^{\forall}(b(y))\ (y : W)$

The notion of quantification deployed by these rules corresponds to what in standard modal logic is known as *actualist quantification* – i.e., the scope of the quantifiers (the variables of which range over *W*) is circumscribed by the objects that "inhabit" *W*. In our setting this means that the players must choose for their moves objects suitable in relation to the hypothetical *y* : *W*, i.e. those functions defined over de domain *W*. This yields a new approach to free logic in general and to dialogical free logic in particular: One can infer *Something is a vampire* from *Nosferatu is a vampire* only if the existential is relativized to the same world *W* upon which *Nosferatu* is dependent. We cannot, for example, infer $b(y) : (\exists x : A)\ Bx(y)\ (y : W)$, neither from the hypothetical $a(z) : Bk(z)\ (z : V)$ – where $k : A$ – nor from the categorical expression $a : Bk$ – where $k : A$. From the dialogical perspective the point is that, because of the formal rule, in order to win **P** can only choose those play objects that **O** has chosen before. Thus, since during the plays involving our example, **O** brings forward $a(z) : Bk(z)\ (z : V)$ in the first example and $a : Bk$, in the second, **P** will not be able to make a suitable choice that yields a copy-cat of those moves of **O**. Under these circumstances, the closest move that **P** can obtain is $a(y) : Bk(y)\ (y : W)$ – that will yield a **P**-win for the first example iff $V = W$ and a win in the second example iff there is no (more) hypothesis. More generally, the following tautological hypothetical implications are valid (there is a winning strategy for **P**) : $b(y) : Bk \rightarrow (\exists x : A)\ Bx\ (y : W)$ and $b(y) : (\forall x : A)Bx \rightarrow Bk\ (y : W)$ only if the formation rules for $Bx : prop$ presuppose a set such that this set is dependent upon *W*.

Notice that this analysis also yields an interpretation of what is happening from the point of view of free logic in the so-called Smuyllan formula when formulated as $z(u) : (\exists x : A)\ (Bx \rightarrow \forall y(y : A)By)\ (u : W)$ that is quite different from the one given by Redmond (2010), Fontaine and Redmond (2011) and Fontaine (2013) (see Appendix II). Indeed, the dynamics involved here is the dynamics triggered by the choices, dependent upon *W*, within the set *A*, not by the changes involving the ontological status of the play objects within *A*.

Now, let us come back to precedent work on dialogical free logic; the point is, to put it bluntly, that the notion of existence in an argument amounts to those commitments to existence that have been brought forward for the sake of the argument at stake. Moreover, since the proponent will copy-cat those existential commitments that have been forward for the sake of the argument, the existential commitments of both sides are hypothetical or, to make use of D. Walton's metaphorical description, those commitments involve plays of *make believe*. Neither the standard logical nor the standard dialogical notation provides a device to express this hypothetical character at the object language level. Now, we know how this can be carried out: quantified propositions are made dependent on sets, the elements of which are yet unknown play-objects. So long as the arguments of the function are unknown, those quantified expressions will continue to be hypothetical and so too their concomitant existential choices.

It is not certain that the usual distinctions, positive, negative, neutral, outer domain free logics apply here. If we were prepared to combine the categorical and the hypothetical levels, we might understand the CTT-approach as describing some kind of supervaluational interpretation of outer domain free logics. However, the comparison is rather analogical, since the CTT-approach does not make use of model theoretic semantics that provides the reference of singular terms. Be that as it may, we must delve into the following questions: What corresponds to the accessibility relation of standard modal logic? What is a set *W*? How do we understand fictional objects such as *Holmes*? The answer to these questions takes us to the next sections.

What are sets W and how are they structured?

Epistemic alternatives, Accessibility and the Dialogical perspective[15]

One way to see the relation between a world W_1 and a world W_2 is to see it as an epistemic alternative, where W_2 is an extension of W_1 in the sense that W_2 adds information in such a way that every proposition that is true under the hypothesis W_1 is also true under the hypothesis W_2. More generally we express this situation in the following way: $d(y) : W_1 (y : W_2)$. Thus, if W_2 is accessible from W_1, then there is a function **f** from W_2 to W_1 (see Ranta, 1994, p. 147). But certainly there might be many such functions that express not only that W_2 from W_1 but also that, say, *V* and *U* are accessible too from W_1, although W_2, *V*, *U* are not accessible between them. The whole yields a tree structure with W_1 as its root. In the dialogical frame the point is that if it is the case that $a(b(y)) : A\ (b(y) : W\ (y : V))$, and $a(x)) : A(x : W)$, then players can bring forward $b(y) : A\ (y : V)$. Interesting is the fact that one way to understand $a(b(y))$ is the variant of the object $a(x)$ when 'transferred' to the world *V*. If the language contains modal operators, the relation between *W* and *V* can also be brought dynamically into the play by the choices of the players as is usual in dialogical modal logic (Rahman and Rückert, 2001); however, for the purpose of the present study of hypothetical objects such operators are not needed.

[15] For the CTT approach to the notion of possible worlds in the present and the following sections we follow Ranta (1991, 1994).

Worlds as Contexts

We still do not know exactly what the sets W are and in what sense they can be understood as expressing the idea of "possible world". Recall that in this context each world W, V, U is a set – where such a set is a hypothesis. What we need to elucidate is both, what the elements of this set are and in what sense the notion of hypothesis captures the idea of possible. Let us start with the latter. From the epistemic point of view, "possible" means different alternatives of adding knowledge provided full knowledge has not been yet achieved. Ranta (1991, p. 78) links this notion of possibility with Husserl's (Cart. Med., p. 62) conception of different ways of completing what I know. Here we are then, in such a frame, possible means that there are (at the disposal of the epistemic agent) several alternative ways of completing not yet achieved full knowledge. Moreover, this means that possible is always an approximation to full knowledge: if the approximation were to end, then possibility would not be any more there, but full knowledge. Possible is that which can always be completed. But how to express this notion formally and how to link it with the dialogical approach? Formally speaking, a possible world is itself a set constituted by a sequence of hypothetical assertions with a dependence defined between them (this structure is called a context). Let us call Γ the sequence that approximates a world, then we have

$$a : \mathrm{A} \text{ in } \Gamma \text{ means } a(x_1, \ldots, x_n) : A(x_1, \ldots, x_n)\ (x_1 : A_1, \ldots, x_n : A_n(x_1, \ldots, x_{n-1}))$$

and something similar applies to: A : set in Γ, $A = B$: set in Γ, and $a = b : A$ in Γ

As mentioned above, if contexts should capture the notion of possible world, it is important that the approximations never end. Thus, as pointed out by Ranta (1991, p. 93), worlds are a kind of limits of sequences of hypothetical assertions further and further specified – without ever reaching full specification. In fact there are two ways to extend a context by adding information to it, and one by reducing uncertainty in it, namely:

(1) By adding a further hypothesis. It may happen that the new proposition, say, A(x1, …, xn) be potentially true in Γ (that is, when the new proposition can be inferred from the original context – then there is growth only in actual knowledge), but it could also add a new piece of information.

(2) By introducing a definition for some variables and achieving in this way a reduction of uncertainty. For instance, the context
$\Gamma = (x_1 : A_1, \ldots\ x_n : A_n)$ is extended to
$\Gamma, x_k = c : A_k$
so that in the new context every occurrence of x_k is substituted by a. The new context is obtained from Γ by removing the hypothesis $x_k : A_k$ by $c(x_1 \ldots x_n)$. Thus the new context is shorter than the original one. Still, this operation furnishes not only the knowledge of the original context but the value of the one variable reduces the uncertainty within the context.

(3) By presenting a new context Δ such that there is function f from Δ to Γ as explained in II.2.1

As observed by Ranta (1994, p. 146-47), (1) and (2) can be seen as special cases of (3). Thus (3) provides the most general case of extension and, at the same time, deploys the constructive meaning of the notion of accessibility. Dialogically speaking, extensions of contexts are to be understood as answers to questions of specification – recall that we are dealing with a fully interpreted language. Assume that one player brings forward the hypothetical that there is a play object for A(y), provided x is a living being, y is a human(x). Then the first kind of extension will the triggered by a question such as is (s)he European or Asian?. The second kind of extension will be triggered by a wh-question (that is a who, what or when question). The third kind of extension can be thought as asking the defender to establish a link between the variables of the first and the new context. Assume for instance that the initial context Γ contains the disjunction $A \vee B$ the hypothetical play object of which is the variable x. Assume further on that the new context Δ contains $y : A$. In such a case the player that claims that Δ is an extension of Γ must produce the definition $L^{\vee}(x) = y : A \vee B$, and something similar must happen in relation to every component of Γ.

Summing up, from the dialogical perspective modality amounts to a dialogue where moves involve questions and answers in relation to underlying contexts.

Let us finish this section by pointing out that, in relation to the second way of adding knowledge by reducing uncertainty, Martin-Löf suggested, in a series of lectures on choice sequences in 1990-1991, that it should be understood as the growth of knowledge in the context of scientific experimentation, where an unknown quantity is given a value but the function is still dependent on other unknown elements. As we will mention in the conclusions, we think that this way of extending a contexts is quite fruitful to understand the role of scientific models within a fictionalist approach to them. This requires reflecting – in the frame so far developed – on what the features of hypothetical entities are. We cannot study here these features in detail – our main aim was to develop a formal frame for the thorough study of this topic – but a brief discussion will show what the issue is about.

Conclusions: Hypothetical objects as fictions and scientific models

Ranta (1994, p. 135) defends the idea that fictional characters represent arbitrary objects of the suitable kind: they are variables (a similar idea was defended by Nelson Goodman). It is the background knowledge of the reader that might extend the context by defining one given variable. When speaking of fictional characters in literature, the point might be controversial, but if we are thinking of scientific models as fictions – as defended by, among others, Godfrey-Smith (2006, 2009) and Frigg (2010), the corresponding hypothetical objects can certainly be seen as variables that in order to be applied require a definition (in the sense of extension by definition as mentioned above) or some other kind of further specification. Certainly, as pointed out already, it is only at the ontological level that fictions and models are on a par, not in their use or role. In this respect, we agree with Giere's (2009) remarks that the use of a scientific model is not the same as the use of a work of fiction and that ignoring such a distinction can even be dangerous[16]. In order to avoid mixing the ontological with the epistemological role, Giere recommends to substitute, in the case of models, the word fiction by a new denomination that also stresses the pragmatics character. Our frame proposes an adequate substitute, namely hypothetical objects. If we follow this path, objects described by models are variables that can only be approximated to a target system. However, the entities

[16] Giere (2009, p. 257-258) recalls in this context the misuse Creationists make of Popper's early remarks that evolutionary theory is not a scientific one.

of models are experimental (hypothetical) objects of some type and thus are not fixed to any object in particular. Let us quote here a passage from Frigg's (2010) paper on *Models and Fiction*:

> But once we acknowledge that these descriptions describe hypothetical systems rather than real target systems, we also have to acknowledge that hypothetical systems are an important part of the theoretical apparatus we employ, and that they therefore have to be included in our analysis of how scientific modelling works. This can, of course, be done in different ways. My suggestion is that these hypothetical systems in fact are the models systems we try to understand, and I therefore I reserve the term 'model system' for the hypothetical physical entities described by the descriptions we use to ground structural claims (Frigg, 2010, p. 259).

The dialogical interpretation adds the interactive component: specifications of the contexts that define a model are the result of questions and answers formulated during the search for growth of knowledge. Relevant to our discussion is Frigg's (2010, p. 17-19, footnote 19) observation that the worlds he has in mind are never complete and that the link between the target system and the model is not one of reference but of a game of make believe. Once more, the analogy to games of make believe should not lead to conflate the use of fictional works and scientific models. Indeed, *anchoring* the hypothetical objects of a context in the context of the target system can be seen as a game of make believe, but his anchoring reduces – to some degree – uncertainty in relation to the target system. A thorough study that differentiates the anchoring (by approximation) of hypothetical objects that constitute scientific models from the anchoring of fictional characters in the belief context of an agent is due. However, the combination of the games of dialogical logic and the CTT approach to possible worlds looks a promising path to explore.

Let us devote our last words to the notion of fiction as artefact. Following Ingarden and Husserl, Thomasson (1999) proposes a realist perspective of the fictional characters, which she calls abstract artefacts. Understanding fictional characters as artefacts means conceiving them as entities dependent on a singular act of creation or birth. Hence they are dependent on an author with whom they maintain a historical link (*historical ontological dependence*) and, at the same time, they are dependent on copies of the literary work where they appear (*generic ontological dependence*). This is what allows them to enter into the linguistic community of readers and continue to exist. Their status of being is continuously restored through these same relationships, that is, they are identifiable through these chains of dependence and, ultimately, susceptible to dying (disappearing) if these relationships are definitively dissolved. These distinctions of Thomasson allow us to think in a bipolar domain: on the one hand, the dependent entities and, on the other hand, the entities on which they depend (which corresponds to the classical distinction between the real and the fictitious). It should be clear that hypothetical objects as defined above are in Thomasson's sense generically dependent objects. Thus, under this perspective the hypothetical object is generically dependent on some arbitrary unknown element of the context G (that is, a variable that instantiates G): this is what generic ultimately means. Historical dependence is not difficult to implement either, once we introduce types for temporality (Ranta, 1994, p. 102-124). Once more, a thorough exploration is due. In fact, it is part of our work in progress, but, so our claim, the frame for such an investigation has been set herewith.

Appendix I: Standard Dialogical Logic[17]

Let L be a first-order language built as is usual upon the propositional connectives, the quantifiers, a denumerable set of individual variables, a denumerable set of individual constants and a denumerable set of predicate symbols (each with a fixed arity).

We extend the language L with two labels **O** and **P**, standing for the players of the game, and the two symbols '!' and '?'. When the identity of the player does not matter, we use variables **X** or **Y** (with **X**≠**Y**). A move is an expression of the form '***X***-*e*', where e is either of the form '!φ' for some sentence φ of L or of the form '?[!φ_1, …, !φ_n]'.

The *particle* (or local) *rules* for standard dialogical games are given on Table 2.

Table 2. Local meaning of standard logical constants.

Previous move	**X** ! φ∧ψ	**X** ! φ∨ψ	**X** ! φ→ψ	**X** ! ¬φ
Challenge	**Y ?** [!φ] or **Y** ? [!ψ]	**Y ?** [! φ, ! ψ]	**Y** ! φ	**Y** ! φ
Defence	**X** ! φ resp. **X** ! ψ	**X** ! φ or **X** ! ψ	**X** ! ψ	– –

Previous move	**X** ! ∀*x*φ	**X** ! ∃*x*φ
Challenge	**Y ?** [!φ(x/a_i)]	**Y** [! φ(x/a_1),…,!φ(x/a_n)]
Defence	**X** ! φ(x/a_i)	**X** ! φ(x/a_i) with 1≤ i ≤n

In this table, the ais are individual constants and φ(x/a_i) denotes the formula obtained by replacing every free occurrence of x in φ by a_i. When a move consists in a question of the form '? [! φ_1,…,!φ_n]', the other player chooses one formula among φ_1,…, φ_n and plays it. We thus distinguish conjunction from disjunction and universal quantification from existential quantification in terms of which player chooses. With conjunction and universal quantification, the challenger chooses which formula he asks for. With disjunction and existential quantification, it is the defender who can choose between various formulas. Notice that there is no defence in the particle rule for negation. Particle rules provide an abstract description of how the game can proceed locally: they specify the way a formula can be challenged and defended according to its main logical constant. In this way the particle rules govern the local level of meaning. Strictly speaking, the expressions occurring in the table above are not actual moves because they feature formula schemata and the players are not specified. Moreover, these rules are indifferent to any particular situations that might occur during the game. For these reasons we say that the description provided by the particle rules is abstract.

[17] The following brief presentation of standard dialogical logic is based on Clerbout (2013). The main original papers on dialogical logic are collected in Lorenzen and Lorenz (1978). For a historical overview see Lorenz (2001). Other papers have been collected more recently in Lorenz (2008, 2010a, 2010b). A detailed account of recent developments can be found in Rahman and Keiff (2005) and Keiff (2009). For the underlying metalogic see Clerbout (2013, 2014). For a textbook presentation: Redmond and Fontaine (2011) and Rückert (2011). For the key role of dialogic in regaining the link between dialectics and logic, see Keiff (2009), Rahman and Keiff (2010). Fiutek *et al.* (2010) study the dialogical approach to belief revision. Clerbout *et al.* (2011) study Jain Logic in the dialogical framework. Popek (2012, p. 223-244) develops a dialogical reconstruction of medieval ***obligationes***. See also Magnier (2013) on dynamic epistemic logic and legal reasoning in a dialogical framework.

Since the players' identities are not specified in these rules, particle rules are symmetric: the rules are the same for the two players. The local meaning being symmetric (in this sense) is one of the greatest strengths of the dialogical approach to meaning. This is in particular the reason why the dialogical approach is immune to a wide range of trivializing connectives such as Prior's *tonk* (see Rahman and Redmond, 2016). The expressions occurring in particle rules are all move schematas. The words "challenge" and "defence" are convenient to name certain moves according to their relation with other moves which can be defined in the following way. Let σ be a sequence of moves. The function p_σ assigns a position to each move in σ, starting with 0. The function F_σ assigns a pair [*m*,*Z*] to certain moves N in σ, where m denotes a position smaller than p_σ(N) and Z is either C or D, standing respectively for "challenge" and "defence". That is, the function F_σ keeps track of the relations of challenge and defence as they are given by the particle rules. Consider for example the following sequence σ:

$$\mathbf{P}\ !\ \varphi\wedge\psi,\ \mathbf{P}\ !\ \chi\wedge\psi,\ \mathbf{O}\ ?\ [\ !\ \varphi],\ \mathrm{P}\ !\ \varphi$$

In this sequence we have for example $p_\sigma(\mathbf{P}\ !\ \chi\wedge\psi) = 1$. A *play* is a legal sequence of moves, i.e., a sequence of moves which observes the game rules. Particle rules are not the only rules which must be observed in this respect. In fact, it can be said that the second kind of rules named structural rules are the ones giving the precise conditions under which a given sentence is a play. The dialogical game for φ, written D(φ), is the set of all plays with φ being the thesis (see the Starting rule below). The structural rules are the following:

SR0 (Starting rule). Let φ be a complex sentence of L and *i*, *j* be positive integers. For every $\zeta\in$D(φ) we have:

- $p\zeta(\ \mathbf{P}\ !\ \varphi)=0$,
- $p\zeta(\ \mathbf{O}\ n{:=}i)=1$,
- $p\zeta(\ \mathbf{P}\ m{:=}j)=2$.

In other words, any play ζ in D(φ) starts with **P** positing φ. We call φ the thesis of both the play and the dialogical game. After that, the Opponent and the Proponent successively choose a positive integer called repetition rank. The role of these integers is to ensure that every play ends after finitely many moves in the way specified by the next structural rule.

SR1 (Classical game-playing rule). Let $\zeta \in$D(φ). For every Min ζ with $p_\zeta(M)>2$ we have $F_\zeta(M)=[m',Z]$ with $m'<p_\zeta(M)$ and $Z\in\{C,D\}$. Let r be the repetition rank of player X and $\zeta\in$D(φ) such that

- the last member of ζ is a **Y**-move,
- M_0 is a **Y**-move of position m_0 in ζ,
- M1,...,Mn are **X**-moves in ζ such that $F_\zeta(M1)=\ldots F_\zeta(Mn)=[m_0,Z]$.

Consider the sequence[18] $\zeta'=\zeta$*N where N is an X-move such that $F_\zeta{}'(N)=[m_0,Z]$. We have $\zeta'\in$D(φ) only if n<r.

The first part of the rule states that every move after the repetition rank choices is either a challenge or a defence. The second part ensures finiteness of

[18] We use ζ*N to denote the sequence obtained by adding move N to the play ζ.

plays by setting the player's repetition rank as the maximum number of times he can challenge or defend against a given move by the other player.

SR2 (Formal rule). Let ψ be an elementary sentence, N be the move **P** ! ψ and M be the move **O** ! ψ. A sequence ζ of moves is a play only if we have: if $N \in \zeta$ then $M \in \zeta$ and $p\zeta(M) < p\zeta(N)$.

That is, the Proponent can play an elementary sentence only if the Opponent has played it previously. The Formal rule is one of the characteristic features of the dialogical approach: other game-based approaches do not have it. Indeed with this rule the dialogical framework comes with an internal account for elementary sentences: an account in terms of interaction only, without depending on metalogical meaning explanations for the non-logical vocabulary. More prominently, this means that the dialogical account does not rely – contrary to Hintikka's GTS games – on the model-theoretical approach to meaning for atomic formulas.

Here is some terminology for the last structural rule in standard dialogical games. A play is called *terminal* when it cannot be extended by further moves in compliance with the rules. We say it is **X**-terminal when the last move in the play is an **X**-move.

SR3 (Winning rule). Player **X** wins the play ζ only if it is **X**-terminal.

Consider for example the following sequences of moves:

P ! $Q(a) \wedge Q(b)$, **O**-n:=1, **P** m:=6, **O**-?[$Q(a)$], **P** ! $Q(a)$
P ! $Q(a) \wedge Q(a)$, **O**-n:=1, **P** m:=12, **O** ! $Q(a)$ **P** ! $Q(a)$

The first one is not a play because it breaks the Formal rule: with his last move, the Proponent plays an elementary sentence which the Opponent has not played beforehand. By contrast, the second sequence is a play in D(**P**! $Qa \rightarrow Qa$). We often use a convenient table notation for plays. For example, we can write this play as follows:

O			**P**	
			!$Q(a) \rightarrow Q(a)$	0
1	n:=1		m:=12	2
3	!$Q(a)$	(0)	!$Q(a)$	4

The numbers in the external columns are the position of the moves in the play. When a move is a challenge, the position of the challenged move is indicated in the internal columns, as with move 3 in this example. Notice that such tables carry the information given by the functions p and F in addition to representing the play itself.

However, when we want to consider several plays together – for example when building a strategy – such tables are not that perspicuous. So we do not use them to deal with dialogical games for which we prefer another perspective. The *extensive form* of the dialogical game D(φ) is simply the tree representation of it, also often called the game-tree. More precisely, the extensive form E_φ of D(φ) is the tree (T,l,S) such that:

(i) Every node t in T is labelled with a move occurring in D(φ)
(ii) l: $T \rightarrow N$
(iii) $S \subseteq T^2$ with:

- There is a unique t_0 (the root) in T such that $l(t_0)=0$, and t_0 is labelled with the thesis of the game,
- For every $t \neq t_0$ there is a unique t' such that t'St,
- For every t and t' in T, if tSt' then $l(t')=l(t)+1$,
- Let $\zeta \in D(\varphi)$ such that $p\zeta(M')=p\zeta(M)+1$. If t and t' are respectively labelled with M and M', then tSt'.

Many dialogical game metalogical results are obtained by leaving the level of rules and plays to move to the level of strategies. Significant among these results are the ones concerning the existence of winning strategies for a player. We will now define these notions and give examples of such results.

A strategy for player **X** in $D(\varphi)$ is a function which assigns an **X**-move M to every non-terminal play ζ having a Y-move as last member such that extending ζ with M results in a play. An X-strategy is winning if playing according to it leads to X's victory no matter how **Y** plays.

Strategies can be considered from the perspective of extensive forms: the extensive form of an ***X***-strategy s in $D(\varphi)$ is the tree-fragment $S_\varphi=(T_s,l_s,S_s)$ of E_φ such that:

(i) The root of S_φ is the root of E_φ,
(ii) Given a node t in E_φ labelled with an ***X***-move, we have $t' \in T_s$ and tS_st' whenever tSt'.
(iii) Given a node t in E_φ labelled with a ***Y***-move and with at least one t' such that tSt', we have a unique s(t) in T_s with $tS_ss(t)$ and s(t) is labelled with the ***X***-move prescribed by s.

Here are some results pertaining to the level of strategies:[19]

- Winning P-strategies and leaves. *Let w be a winning P-strategy in D(φ). Then every leaf in the extensive form Wφ of w is labelled with a* **P** *elementary sentence.*
- Determinacy. *There is a winning X-strategy in D(φ) if and only if there is no winning* ***Y****-strategy in D(φ).*
- Soundness and Completeness of Tableaux. *Consider first-order tableaux and first-order dialogical games. There is a tableau proof for φ if and only if there is a winning* **P***-strategy in D(φ).*

The fact that existence of a winning **P**-strategy coincides with validity (*there is a winning* **P***-strategy in D(φ) if and only if φ is valid*) follows from the soundness and completeness of the tableau method with respect to model-theoretical semantics.

Regarding several results, extensive forms of strategies have key parts: one of the parts of a winning strategy, called the *core* of the strategy, is actually that on which one works when considering translation algorithms such as the procedures. The basic idea behind the notion of core is to get rid of redundant information (for example, different orders of moves) which we find in extensive forms of strategies (see Clerbout and Rahman, 2015).

[19] These results are proven, together with others, in Clerbout (2014).

Appendix II: The Dialogical Approach to Free Logic

The existential presuppositions related to singular terms of standard classical logic can be made explicit through the analysis of the following axioms (that are related to the obvious inferential rules):

$\forall x\varphi \rightarrow \varphi[x/ki]$ (Specification)
$\varphi[x/ki] \rightarrow \exists x\varphi$ (Particularisation)

Any logic that includes these two principles is ontologically committed with respect to its singular terms. One way to free oneself from this commitment is to maintain the reach (classical) of the quantifiers, but broaden the referentiality of the singular terms that lead to a double domain (*outer* and *inner*). This would collapse the two principles and validate the following definition of Hintikka (1966): $E!(a) = \text{def}\ \exists x(x = a)$. By means of the predicate $E!$ then the presupposition of existence in the assertions is made explicit. You may also think of a semantic with Meinongian quantifiers, which would yield – following Priest (2005, p. 14-15) – the following:

"All existent things are such that…" : $\forall x\alpha[x] =\text{df}\ \Lambda x(E!x \rightarrow \alpha[x])$;
"There exists something such that…" : $\exists x\alpha[x] =\text{df}\ \Sigma x(E!x \wedge \alpha[x])$.

Notice that making these presuppositions explicit forces us to confront the philosophical difficulties of considering existence as a certain type of property of things. For the pragmatic approach that we have chosen, the meaning of the quantifiers results from the interactions between an action and a proposition. That is, between the action of choosing a constant of substitution and the proposition that results from this action – an interesting antecedent can be found in the work of Jaskowski (1934), who proposes to make use of assumptions that express the introduction of singular terms at the object-language.

The first developments on dialogical free logic (such as Rahman *et al.*, 1990) regulate the interaction between choices and propositions by means of a special structural rule called the "rule of introduction".

Rule of Introduction: Let us say that singular term *ki* played by **X** has been introduced if: X brings forward $\varphi[x/k_i]$ to defend an existential expression of the form $\exists x\varphi$, or if X attacks $\forall x\varphi$ con < ?-x/k_i >, where k_i does not occur in the play before. Only **O** can introduce singular terms.

With this rule both – specification and particularisation – are invalidated.

O			P	
			$Ak_1 \rightarrow \exists xAx$	0
1	Ak_1	0	$\exists xAx$	2
3	?-$\exists$	2		

O			P	
			$\forall xAx \rightarrow Ak_1$	0
1	$\forall xAx$	0		

One of the most remarkable consequences of this logic is that all the formulas that start with an existential quantifier are invalid. Redmond and Fontaine remediated this shortcoming (in Redmond, 2010; Fontaine and Redmond, 2011; Fontaine, 2013) with what has been called a *dynamic free logic* that amounts to the following reformulation of the structural rule for free logic:

Let us say that singular term ki played by **X** has been introduced if: **X** brings forward $\varphi[x/k_i]$ to defend an existential expression of the form $\exists x\varphi$, or if **X** attacks $\forall x\varphi$ with < ?-x/k_i >, where k_i does not occur in the play before. **P** can make use of singular term only iff this singular term is absolutely new in the play or has been introduced by **O** before.

Appendix III: The Dialogical Approach to CTT[20]

The dialogical approach to CTT starts with play-objects rather than with proof-objects. Proof-objects are a subset of the former, namely those that constitute a winning strategy. Before delving into the details about play-objects, let us first discuss the issue of the formation of expressions, and in particular of propositions, in the context of dialogical logic.

The Formation of Propositions

In standard dialogical systems there is a presupposition that the players use well-formed formulas (wff's). One can check well-formedness at will, but only via the usual metareasoning by which one checks that the formula indeed observes the definition of wff. The first addendum we want to make is to allow players to question the status of expressions, in particular to question the status of something as actually standing for a proposition. Thus we start with rules giving a dialogical explanation of the *formation* of propositions. These are local rules added to the particle rules which give the local meaning of logical constants (see next section).

Let us make a remark before displaying the formation rules. Because the dialogical theory of meaning is based on argumentative interaction, dialogues feature expressions which are not posits or sentences. They also feature requests used as challenges, as illustrated by the formation rules below and the particle rules in the next section. Now, by the *No entity without type* principle, the type of these actions, which type we shall write "formation-request", should be specified during a dialogue (see Table 3).[21]

Table 3. The formation-rules.

Posit	Challenge [when different challenges are possible, the challenger chooses]	Defence
X ! Γ : set	**Y** $?_{can}\ \Gamma$ or **Y** $?_{gen}\ \Gamma$ or **Y** $?_{eq}\ \Gamma$	**X** ! $a_1 : \Gamma$, **X** ! $a_2 : \Gamma$, … (**X** gives the canonical elements of Γ) **X** ! $a_i : \Gamma \Rightarrow a_j : \Gamma$ (**X** provides a generation method for Γ) (**X** gives the equality rule for Γ*)

[20] The present overview on the dialogical approach to CTT is based on Rahman and Clerbout (2013, 2014) and Clerbout and Rahman (2015).
[21] Such a move could be written as "$?_{F1}$: formation-request".

X ! $\varphi\vee\psi$: prop	**Y** $?_{F\vee 1}$ or **Y** $?_{F\vee 2}$	**X** ! φ : prop **X** ! ψ : prop
X ! $\varphi\wedge\psi$: prop	**Y** $?_{F\wedge 1}$ or **Y** $?_{F\wedge 2}$	**X** ! φ : prop **X** ! ψ : prop
X ! $\varphi\rightarrow\psi$: prop	**Y** $?_{F\rightarrow 1}$ or **Y** $?_{F\rightarrow 2}$	**X** ! φ : prop **X** ! ψ : prop
X ! $(\forall x : A)\ \varphi(x)$: prop	**Y** $?_{F\forall 1}$ or **Y** $?_{F\forall 2}$	**X** ! A : set **X** ! $\varphi(x)$: prop $(x : A)$
X ! $(\exists x : A)\ \varphi(x)$: prop	**Y** $?_{F\exists 1}$ or **Y** $?_{F\exists 2}$	**X** ! A : set **X** ! $\varphi(x)$: prop $(x : A)$
X ! $B(k)$: prop (for atomic B)	**Y** $?_F$	**X** *sic* (n) (**X** indicates that **Y** posited it at move n)
X ! $\perp$: prop	–	–

Note: (*) Equality rules are presented in the next section.

By definition the *falsum* symbol $\perp$ is of type prop. Therefore the formation of a posit of the form $\perp$ cannot be challenged.

The next rule is not a formation rule per se but rather a substitution rule.[22]

Posit-substitution

There are two cases in which **Y** can ask **X** to make a substitution in the context $x_i : A_i$. The first one is when in a standard play a variable (or a list of variables) occurs in a posit with a proviso. Then the challenger posits an instantiation of the proviso:

Posit	Challenge	Defence
X ! $\pi(x_1, \ldots, x_n)\ (x_i : A_i)$	**Y** ! $\tau_1 : A_1, \ldots, \tau_n : A_n$	**X** ! $\pi(\tau_1 \ldots \tau_n)$

The second case is in a formation-play. In such a play the challenger simply posits the whole assumption as in Move 7 of the example below:

Posit	Challenge	Defence
X ! $\pi(\tau_1, \ldots, \tau_n)\ (\tau_i : A_i)$	**Y** ! $\tau_1 : A_1, \ldots, \tau_n : A_n$	**X** ! $\pi(\tau_1, \ldots, \tau_n)$

Play objects. The idea is now to design dialogical games in which the players' posits are of the form "$p : \varphi$" and acquire their meaning in the way they are used in the game – i.e., how they are challenged and defended. This requires, among other things, analysing the form of a given play-object p, which depends on φ, and how a play-object can be obtained from other, simpler, play-objects. The standard dialogical semantics for logical constants gives us the needed information for this purpose. The main logical constant of the expression at stake provides the basic information as to what a play-object for that expression consists of:

[22] It is an application of the original rule from CTT given in Ranta (1994, p. 30).

A play for **X** $\varphi\vee\psi$ is obtained from two plays p_1 and p_2, where p_1 is a play for **X** φ and p_2 is a play for X ψ. According to the particle rule for disjunction, it is the player **X** who can switch from p_1 to p_2 and vice-versa.

A play for **X** $\varphi\wedge\psi$ is obtained similarly, except that it is the player Y who can switch from p_1 to p_2.

A play for **X** $\varphi \to \psi$ is obtained from two plays p_1 and p_2, where p_1 is a play for **Y** φ and p_2 is a play for X ψ. It is the player **X** who can switch from p_1 to p_2.

The standard dialogical particle rule for negation rests on the interpretation of $\neg\varphi$ as an abbreviation for $\varphi \to \bot$, although it is usually left implicit. It follows that a play for **X** $\neg\varphi$ is also of the form of a material implication, where p_1 is a play for **Y** φ and p_2 is a play for **X** $\bot$, and where **X** can switch from p_1 to p_2.

As for quantifiers, we are dealing with quantifiers for which the type of the bound variable is always specified by a set. This brings us to Table 5.

Table 5. Local meaning of logical constants in a CTT-setting.

Posit	**Challenge**	**Defence**
X ! φ (where no play-object has been specified for φ)	**Y** ? play-object	**X** ! $p : \varphi$
X ! $p : \varphi\vee\psi$	**Y** ?[φ/ψ]	**X** ! $L^{\vee}(p) : \varphi$ Or **X** ! $R^{\vee}(p) : \psi$ **[the defender has the choice]**
X ! $p : \varphi\wedge\psi$	**Y** $?_L$ Or **Y** $?_R$ **[the challenger has the choice]**	**X** ! $L^{\wedge}(p) : \varphi$ respectively **X** ! $R^{\wedge}(p) : \psi$
X ! $p : \varphi \to \psi$	**Y** ! $L^{\to}(p) : \varphi$	**X** ! $R^{\to}(p) : \psi$
X ! $p : \neg\varphi$	**Y** $L^{\bot}(p) : \varphi$	**X** $R^{\bot}(p) : \bot$
X ! $p : (\exists x : A)\varphi$	**Y** $?_L$ Or **Y** $?_R$ **[the challenger has the choice]**	**X** ! $L^{\exists}(p) : A$ Respectively **X** ! $R^{\exists}(p) : \varphi(L(p))$
	Y $?_{prop}$	**X** ! $(\forall x : A)\varphi$: prop
X ! $p : (\forall x : A)\varphi$	**Y** ! $L^{\forall}(p) : A$	**X** ! $R^{\forall}(p) : \varphi(L(p))$
X ! $p : B(k)$ (for atomic B)	**Y** ?	**X** sic (n) (X indicates that Y posited it at move n)

It may happen that the form of a play-object is not explicit at first. In such cases we deal with expressions of the form, e.g., "$p : \varphi\wedge\psi$". In the relevant challenges and defences, we then use expressions such as $L\wedge(p)$ and $R\wedge(p)$ used in our example. We call these expressions *instructions*. Their respective interpretations are "take the left part of p" and "take the right part of p". In instructions we indicate the logical constant at stake: it keeps the formulations explicit enough, in particular in the case of embedded instructions. We must also keep in mind the important differences between play-objects depending on the logical constant that is used. Consider for example the case of conjunction and disjunction:

- A play-object p for a disjunction is composed by two play-objects, but each of them constitutes a sufficient play-object for the disjunction. Moreover it is the defender who makes the choice between $L^{\vee}(p)$ and $R^{\vee}(p)$.
- A play-object p for a conjunction is also composed by two play-objects, but this time the two of them are necessary to constitute the one for the conjunction. It is then the challenger's privilege to ask for either or both (provided the other rules allow him to do so).

Accordingly, $L^{\wedge}(p)$ and $L^{\vee}(p)$, say, are actually different things and the notation takes that into account.

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