To be (something) or not to be: Existence and predication in Aristotle's logic and metaphysics¹

Ser (algo) ou não ser: existência e predicação na lógica e na metafísica de Aristóteles

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ABSTRACT: This paper offers a new interpretation of Aristotle's logical system that allows us to break away from a presumably inescapable dilemma inherent to any attempt of justification from a strictly formal standpoint; namely, either the system *stands apart* from any standard system of mathematical logic — propositional logic, class theory, quantification theory, etc. — or it is *lacking in consistency*. While some other attempts have been made that seem to avoid this dilemma, they have always resulted in an intolerably strong restriction of the Semantics underpinning the Aristotelian system. These are, therefore, unsatisfactory solutions that undermine some of the key principles of Aristotelian metaphysics. One by-product of the solution presented here is a reappraisal of some elements in Aristotle's theory of Being that breaks down several contemporary misconceptions on the subject.

Key words: logic, Aristotle, justification, mathematical logic.

Resumo: Este artigo oferece uma nova interpretação do sistema lógico de Aristóteles que nos permite tomar distância de um dilema supostamente sem escapatória inerente a qualquer tentativa de justificação desde um ponto de partida estritamente formal; mais especificamente, ou o sistema se identifica com algum sistema lógico-matemático – lógica proposicional, teoria de classes, teoria da quantificação, etc. – ou é considerado inconsistente. Embora tenham sido feitas outras tentativas que parecem evitar este dilema, sempre resultaram numa restrição intoleravelmente forte à Semântica que dá sustentação ao

¹Thanks to David Gutiérrez and Bianca McKell for their invaluable work of translation and edition. ²Universidad de Barcelona. Facultad de Filosofia. Montalegre 6-8, 08001, Barcelona, España. E-mail: candel@ub.edu. sistema aristotélico. Estas são, consequentemente, soluções insatisfatórias que minam alguns princípios chaves da metafísica aristotélica. Uma das soluções propostas aqui é uma revalorização de alguns elementos da teoria aristotélica do Ser que derruba alguns mal-entendidos contemporâneos sobre o tema.

Palavras-chave: lógica, Aristóteles, justificação, lógica matemática.

Introduction: Have we to lose the philosophical soul in order to conquer the world of formalism?

When the modern development of Logic, as from Boole onwards, made evident the insufficiently formal nature of classical Logic in general and of Aristotelian Logic in particular, an iconoclastic, lasting obsession took over the scholars in this field (at least until after the Second World War). An attitude became commonplace amongst logicians that could be considered as the opposite of the Hippocratic oath among doctors; namely, a profession of anti-Aristotelian faith as a guarantee of scientific respectability for any logical research worthy of this name.

It was not until the publication of Jan Łukasiewicz's pioneering work (1951-1957) that such global disqualification started to be seen as an exaggeration.³ Aristotelian Logic, having lost its aura of a universally valid system, gradually left also its undeserved place at the bottom of the scientific value scale and started to be regarded as a specific case (or as one of limited validity) within the whole set of possible logical systems (more precisely, as an *application* of a given formal system).

Once this is conceded, a closely related question arises: If Aristotelian Logic is, in fact, an application of a formal system, what particular formal system? Łukasiewicz, referring to its most important formal core, Syllogistic (*Analytics* in Aristotle's words), thinks it is a peculiar (implicit) application of propositional Logic in which syllogisms work as true material implications whose antecedent is made up of the conjunction of the premises and whose consequent is made up of the conclusion.⁴

Łukasiewicz's interpretation, followed by Bocheński (1956-1962) and, with several nuances, by Patzig (1969), was questioned by important scholars such as Kneale and Kneale (1962) and Prior (1962). It has also been refuted (incontestably, in my view) on several key points by Rose (1968), Sainati (1968), Granger (1970), Smiley (1973) and Corcoran (1974a), among others.

Łukasiewicz's approach is no doubt a bona fide move. Russell (1956), Strawson (1971) and Quine (1982) apparently proved, on grounds we will later look into, the impossibility of interpreting the propositions — and consequently the deductions (syllogisms) — of Aristotelian Logic in the formal language of first order predicate Logic and even in the language of the Logic of classes. Therefore, assuming that the underlying Logic of the Aristotelian system is propositional Logic, or truth-functional calculus (a fact of which Aristotle was presumably not aware; although, always according

³ Actually, Łukasiewicz also criticizes Aristotelian Logic from the standpoint of Modern Logic. He saves, though, its essential core and at the same time distinguishes it from the so-called traditional Logic (mostly Scholastic) that, in his opinion, was nothing but a hybrid between properly Aristotelian elements and the Stoic propositional Logic.

⁴ The material implication (or the 'conditional', in Quine's terminology) is valid, too, whenever both the antecedent and the consequent are false. This eventuality is irrelevant for Aristotle, whose aim was to provide science with an infallible deductive method capable of giving shape to the demonstration of *true* propositions.

to Łukasiewicz, he designed some elements of his theory as if he was familiar with some of the principles of such Logic), the Polish author suggests that the Aristotelian system should be accepted as a special logical language whose variables represent universal terms (in the Aristotelian traditional sense) and whose logical functors are the four connectors traditionally symbolized with the letters A (universal affirmative), E (universal negative), I (particular affirmative) and O (particular negative). In the face of criticisms from the majority of modern logicians who consider invalid some of the relations established by Aristotle between propositions in virtue of these four functors,⁵ Łukasiewicz accepts those relations by way of previous definitions upon which he builds his own interpretative scheme in terms, as mentioned above, of propositional Logic (thus superimposing the functors of propositional calculus on the Aristotelian functors).⁶ The cost he bears, or makes Aristotle bear, for this redemption of his logical system (at least as far as Syllogistic is concerned) is simply giving up any attempt of coherent logical analysis of the propositions which constitute syllogisms. This price is doubtless too high, for Aristotle does indeed base his theory of reasoning (or Syllogistic) upon a theory of simple propositions which has, furthermore, profound philosophical implications.⁷ But let us see why Łukasiewicz thinks he must proceed as he does.

Universal versus existential

In chapters seven and eight of the brief treaty, *De Interpretatione* (hereinafter Int.),⁸ Aristotle shows the forms that can be taken by simple propositions made up of a predicate-term (always universal; that is, "referable to many individuals," as defined by him at the beginning of chapter seven) and a subject-term, which may be either singular or universal. When the latter applies, the universal subject may also "be taken universally," which will give rise to propositions with a quantification such as "every X..." It may also be taken non-universally,⁹ which will be expressed by a quantification such as "some X..." What is important is the author's insistence on differentiating the *implicit* universality — which inheres certain terms (basically the traditionally called "common names," as well as adjectives) — from the universality made explicit in propositions by means of the determiners every and none. Moreover, the author considers these determiners to be exclusively applicable to the subject-term, since he thinks it inappropriate to have a proposition such as "Every H is every B," exemplified by the obviously false "Every man is every animal."¹⁰ This clear differentiation between, on one hand, potential or implicit semantic universality and, on the other, an explicit syntactic mark of universal application does not really fit the theory (which has become

⁵ For instance, the presumed implication of *I* by *A* or the incompatibility between *A* and *E*.

⁶ With the help, moreover, of four axioms constituted by the Aristotelian syllogistic modes *Barbara* and *Datisi*, and by two identity principles (never mentioned, directly or indirectly, by Aristotle) expressed in the propositions *XaX* and *XiX*, where *a* and *i* are, of course, the homonymous functors of the Aristotelian system.

⁷ One of the merits attributed by Łukasiewicz (1951-1957, p. 6) to the Aristotelian Syllogistic is that it is entirely exempt from any contamination from philosophical elements allegedly strange to its purely formal nature.

⁸ Our quotations are based upon the standard Oxford translation, edited by J. Barnes, (Aristotle, 1984).

⁹ In *Analytics,* of a subject taken non-universally, it is said that it is "partially" taken. Hence its traditional designation as "particular."

¹⁰ As Aristotle states it, "there cannot be an affirmation in which a universal is predicated universally of the subject" (*De interpretatione*, 17b14-15). His starting point seems to be that the predicate always has a broader extension than the subject does. It does not hold, however, when the predicate is the *definition* of the subject, for then both are coextensive. So, for instance, he would have had no reason to dismiss expressions such as "Every triangle is every portion of a plane limited by three straight lines." Nonetheless, the *Organon's* author considers here the general case of predication, in which the predicate is a simple term and not a combination of terms as definitions are.

predominant as from Łukasiewicz) according to which in Aristotle's Logic there is no equivalent to the contemporary notion of quantification.

But the main interest of the author of *Int.* seems to be to establish the relationship between the four possible types of "quantified"¹¹ propositions from the standpoint of their *truth value* (i.e. from the truth-functional standpoint typical of propositional Logic). To this end, he arranges the four proposition-types by pairs of negations in the following order:

A: Every human being (is) white. /vs/O: Not every human being (is) white. I: Some human being is white. /vs/ E: No human being (is) white.¹²

The criteria applied to establish this scheme are the above-mentioned (i.e. taking, or not, subject-term in its whole extension — "universal taken universally" versus "universal taken non-universally" — and the affirmative or negative statement of the predicate's belonging to the subject). These four features, mutually opposed in pairs, allow establishing different types of opposition (*antikeísthai*) between the statements containing them.

The fact is that in Aristotelian Logic the opposition relationships amount to the truth functions of propositional Logic. As the purpose of the above alignment is to show the opposition between different simple statements made up of the same terms, Aristotle takes into account nothing resembling the truth functions later known (by the Stoics) as conjunction and disjunction.¹³ But he does really consider in Int., explicitly and in extenso, the truth function of negation (apóphasis) and, with less detail in Prior Analytics (hereinafter Pr.An.), that of implication (akoloúthesis). Now, due to the different quantification of these propositions, the negation can be either total or partial.¹⁴ Total negation occurs between the affirmative universal and the negative universal and is called contrariness (enantiótes); we will represent it as $A \mid E$. Partial negation occurs between universals and opposite sign particulars (affirmative versus negative) and is called contradiction (antiphasis); we will represent it as A+O, E+I. The fact, which is at first sight surprising, that Aristotle holds contrariness opposition (instead of contradiction) as being total can only be explained, as we shall see, if we admit as logically sound all the truth functions ascribed by Aristotle to the aforementioned relationships. Łukasiewicz's axiomatic interpretation, for example, which asserts its fidelity to the Aristotelian text, has not been able to tell about this point. Implication is, in its turn, the relationship between universals and particulars of the same sign (both affirmative and negative); we represent it as $A \rightarrow I$, $E \rightarrow O$.

In order to illustrate these three relationships, we can put them in the form of the following truth-functional tables.

¹¹ For comfort, and even before deciding on the legitimacy of its application to Aristotle's Logic, we will provisionally use this term when referring to those propositions introduced by *every*, *none*, & *c*.

¹² In De interpretatione, Aristotle arranges these statements as A, O, E, I. Nonetheless, an allusion to "diagonally (diametrically) opposed" statements at 19b35, plus an explicit arrangement such as ours (which appears at 18a4-6), clea rly shows that the propositions are introduced clockwise. Moreover, it is noticed that the author had in mind (and, surely, in a print accompanying the written text or the oral exposition, cf. 19b26-29) a scheme such as the one we propose here: The universal affirmative placed at the left upper angle, its negation (the particular negative) at the right upper angle, the universal negative at the right lower angle and its negation (the particular affirmative) at the left lower angle. No doubt this scheme was the origin of the tetragon of the Aristotelian tradition, although here, at least as from Ammonius' comment (In de interpr. 75v) onwards, the universals A-E and the particulars I-O are placed from left to right and from top to bottom. In our English version we parenthesize the terms (as is the case here with is, is not) whenever they do not explicitly appear in Greek. ¹³ For this reason, a propositional Logic *sensu stricto* cannot be ascribed to Aristotle as it is ascribed to the Stoics. ¹⁴ The notion of "total negation" is actually expressed as "total falsity," for instance at 54a2-18. Now, it is obvious that every false statement can be considered as the negation of a true one.

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Contradiction		
A/E ¹⁵	O/I	A+O/E+I
V	F	V
F	V	V
F	F	F
V	V	F

Contrariness

Α	E	A ⊣E ¹⁶
V	F	V
F	V	V
F	F	V
V	V	F

Implication

A/E	I/O	A→I/E→O
V	V	V
F	F	V
F	V	V
V	F	F

We could also add the relationship called, since Ammonius, *subcontrariness* (*hypenantiótes*). It was left unnamed by Aristotle and, unlike contrariness, allows for the simultaneous truth and excludes the simultaneous falsity of the propositions so correlated (the particular ones). Such a feature prompts us to call it *compatibility* relationship; we will symbolise it as *I*—O, its truth-functional table reading as follows:

Subcontrariness		
I	0	I—0
V	F	V
F	V	V
V	V	V
F	F	F

¹⁵ When two letters separated by a slash mark appear in the two first columns corresponding to the propositions that are terms of the opposition, it must be understood that the relation goes between the propositions on the same side of the slash in each column and never in a cross-direction — thus, in this case, between A-O and E-I. ¹⁶ As it can be seen, contraries *totally* deny each other in the sense that a statement negates about the *totality* of the semantic field of the subject what its opposite affirms about that very same totality. They are (to say it like Aristotle does) "diametrically opposed." Their mutual "distance" is, therefore, maximal. That is why they admit an "intermediate" truth between the extremes they represent, although they can be both false. Given these characteristics, it would not be nonsensical to designate their mutual relation as one of *incompatibility*, in opposition to the mutual *compatibility* of particulars. The contradictories, in contrast, deny or affirm, respectively, just a *part* of what is denied or affirmed by their opposite. They are, so to say, "minimally" opposed. But also because each one's semantic field begins where that of the other ends, they do not admit an intermediate (principle of *tertio excluso*). Having this in mind, it is easily understood why Aristotle, in his refutation proofs of the syllogistic non-concluding modes (i.e. those that admit several different conclusions), limits himself to show that in every one of them the very same formal structure of the premises can lead to *contrary* (i.e. *totally opposed*) conclusions. This includes, *a fortiori*, every other possible (intermediate) conclusion.



All these relationships are obviously symmetrical except for $A \rightarrow I$, $E \rightarrow O$. It is equally obvious (as pointed out by Russell, Kneale and Kneale, Quine, etc.) that if we interpret the propositions A, E, I, O in the way that is usual within the Logic of first order predicates — with a clear-cut opposition between universal (or hypothetical) quantification, on the one hand, and particular (or existential) quantification, on the other — we can only hold in its integrity the truth-functional table of contradiction. For we would then have:

(1) For A (Every human being is white).	∀x(Hx→Bx)
(2) For E (No human being is white).	∀x(Hx→~Bx)
(3) For I (Some human being is white).	∃x(Hx·Bx)
(4) For O (Some human being is not white).	∃x(Hx·~Bx)

But if, according to De Morgan's laws, we write:

(1 bis) For A (Every human being is white).	~∃x(Hx·~Bx)
(2 bis) For E (No human being is white).	~∃x(Hx·Bx)

it turns out, by virtue of the properties of conjunction, that both A and E could be true if only $\sim \exists x(Hx)$ were true:

That is to say, were there no human being, both *A* and *E* would be simultaneously true, against the emphatic Aristotelian assertion that they can only be simultaneously false. Moreover, it is also clear that $\exists x(Hx \cdot Bx)$ cannot at any rate follow from $\neg \exists x(Hx \cdot \neg Bx)$; similarly, from $\neg \exists x(Hx \cdot Bx)$ no one could possibly conclude $\exists x(Hx \cdot \neg Bx)$ (as it should be the case in virtue of the laws of implication). Indeed, as previously shown, both universal propositions could be true in the case that $\neg \exists x(Hx)$, which precludes any possibility of inferring from them, respectively, $\exists x(Hx \cdot Bx)$ or $\exists x(Hx \cdot \neg Bx)$, as both expressions convey necessarily the truth of $\exists x(Hx)$. Were that the case, we would have (except for contradiction) truth-functional tables completely different from those shown above:

Α	E	AHE
V	F	V
F	V	V
F	F	V
V	V	V
	ſ	
A/E	I/O	A→I/E→O
V	V	V
F	F	V
F	V	V
V	F	V
	1	·
I	0	I—0
V	F	V
F	V	V
F	F	V
V	V	V

It is obvious that every difference between the three relationships is abolished, for all of them have exactly the same truth-values. Furthermore, these values are reduced to one (V) for every value of the correlated propositions, so these "functions" lose any usefulness to propositional calculus.

Such a result is even more disappointing insofar as the above proposed formulæ, $\exists x(Hx \cdot Bx)$ (*I*) and $\neg \exists x(Hx \cdot Bx)$ (*E*), reveal the sound base of the so-called *conversion laws* profusely used by Aristotle in his Syllogistic as a tool to "improve" the modes of the second and third figures by transforming them into modes of the first figure. These laws state that both the universal negative and the particular affirmative propositions can be inverted by exchanging (*converting*, according to the traditional terminology) the positions of the subject-term and the predicate-term with the truth-value of the proposition remaining the same.

The only possibility to get rid of this obstacle seems to be — as suggested by different authors, Strawson (1971) among them, but only to refute the solution or to show its artificiality — to bind the variable, at least for the subject-term,¹⁷ with an existential quantifier, even in universal propositions, so as to have the following result:

(5) For A (Every human being is white.)	~∃x(Hx·~Bx)·∃x(Hx)
(6) For E (No human being is white.)	~∃x(Hx·Bx)·∃x(Hx)

This step allows us to rescue the Aristotelian concept of the relationships of contrariness, $A \mid E$, and implication, $A \rightarrow I$, $E \rightarrow O$, but at the unaffordable cost of ruining the most crucial relationship of contradiction, $A \mid O$, $E \mid I$, which is the basis of the whole logical building of *Organon*, as well as the convertibility of *E*, the cornerstone of Syllogistic.

Indeed, if A is interpreted as in (5), its negation (i.e., its contradiction) does not become simply O interpreted as in (4), namely $\exists x(Hx \sim Bx)$, but is interpreted instead as the disjunction:

(7) $\exists x(Hx \sim Bx)v \sim \exists x(Hx)$

These formulæ preserve the truth functions of contradiction (negation), but alter the initial interpretative scheme.

Much more serious is the fate of the other pair of contradictory propositions: If *E* is interpreted as in (6), its contradiction (*I*) will cease to be just (3), i.e. $\exists x(Hx \cdot Bx)$, but will become the semantically aberrant and abhorrent disjunction:

(8)
$$\exists x(Hx \cdot Bx)v \sim \exists x(Hx)$$

(Who would be ready to accept that the proposition "there is some white human" could possibly mean that "there is no human being at all?")

And, as a matter of fact, if for both statements we maintain (3) and (4), respectively, the relationship each one of them has with the contrary universal ceases to be contradictory and becomes one of contrariety (*sensu aristotelico*).

If $\exists x(Hx)$ is false, then both (A), $\neg \exists x(Hx \cdot \neg Bx) \cdot \exists x(Hx)$, and (O), $\exists x(Hx \cdot \neg Bx)$, will be so. But, still, their simultaneous truth is impossible given the obvious incompatibility of $\neg \exists x(Hx \cdot \neg Bx)$ with $\exists x(Hx \cdot \neg Bx)$. This works analogously for (E), $\neg \exists x(Hx \cdot Bx) \cdot \exists x(Hx)$, and (I), $\exists x(Hx \cdot Bx)$.

¹⁷ Kneale and Kneale (1962, p. 60f) consider that a reference should be presupposed, too, for the predicate to the end of saving both E's convertibility and Syllogistic in general.

In closing, the formula of (*E*), $\sim \exists x(Hx \cdot Bx) \cdot \exists x(Hx)$, by no means follows from its converse $\sim \exists x(Bx \cdot Hx) \cdot \exists x(Bx)$, as Aristotle would require. This is so because although the first member of this formula, $\sim \exists x(Bx \cdot Hx)$, is equivalent to the correspondent of the first (on the grounds of the conjunction commutative property), this equivalence never occurs between the second members, $\exists x(Hx)$ and $\exists x(Bx)$.

If we take this path in our trip through Aristotelian Logic, the baggage of the Russellian Logic of predicates is of no use. Nor is (as some authors propose) that of the Logic of classes with exclusion of the null class, as can be seen on the basis of an elementary analogy.

Now it is plain why Jan Łukasiewicz refused to use both formalisms and denied, by the same token, the existence of genuine quantifiers in Aristotelian Logic. Though, it is not easy to understand why he did not find an alternative interpretation other than the contrived appeal to propositional calculus.

There certainly are other ways out of the impasse.

Alternative solutions

One alternative solution was proposed by Granger (1970), according to whom Aristotelian Syllogistic is a set of *metalinguistic*¹⁸ rules of *semantic* consequence or inference in which only the terms and the propositions they form are found in the object language domain. In addition, within this set of rules the syllogistic modes and figures, as well as the procedures through which they transform into each other — especially conversion and *exposition* (*ékthesis*) — are but meta-reasonings that lie in the determination of correspondences between terms and propositions, on the one hand, and certain semantic models, on the other.¹⁹ The elements of propositional calculus that assuredly appear there belong to the metalogical level and not, as Łukasiewicz holds, to the object language level. The formulation technique of this metalinguistic analysis, used by Granger to symbolize the terms and their relationships of mutual, total or partial inclusion or exclusion, is that of the so-called "Euler's circles,"²⁰ always regarded as non-void term representations.

Sainati (vid. Menne and Öffenberger, 1995, p. 121) holds an analogous position: "Qua syllogistic deduction *rules*, Aristotelian formulæ are not *laws* possessing a 'formal' truth of their own and independent of the terms' value. [...] Logic presents itself to Aristotle not as a formal science but as the metatheory of a certain non-formal object language."²¹

Kneale and Kneale (1962) solution²² states that all the terms used by Aristotle in his formulæ are non-void, reference-endowed terms. It allows a reformulation of the propositions studied by him through the ordinary formalisms of predicate Logic — i.e. with the above formulæ (1 bis), (2 bis), (3) and (4) — but introduces therewith such a strong restriction to the interpretation of those formalisms that in practice it amounts to the suppression of every truly universal quantification.

¹⁸ That is in sharp contrast with Łukasiewicz's interpretation, as well as with the majority of the previous and subsequent attempts to see in Syllogistic a self-contained logical calculus in which both functors and variables would be at the same level of the discourse.

¹⁹ Vid. our Introduction to the Spanish edition of *Prior Analytics* (Aristóteles, 1988, p. 88): "By means of the syllogism, Aristotle attempts to create a *conviction mechanism* that, beyond the Semantics of concrete terms (although both the linking rules and the rules of transformation of some links into others are, at the end, semantic rules), by virtue of its own structure, of its *figure*, makes evident the concatenation between the extremes." ²⁰ As far as we know, Leibniz was the first author who made use of this technique of schematisation of logical relationships. Euler's circles should not be confused with Venn's diagrams, which are much more precise but difficult to adapt to the Aristotelian proposition representation due to its' supposed lack of void terms. ²¹ Translated from the German version). To Sainati, the terms are the only elements belonging to the object

language — even the propositional functors (A, E, I, O) are metalinguistic in nature. We share this opinion. ²² See note 17 *supra*.

Finally, Corcoran's (1974b, p. 92f) proposal is probably the one that has best reconciled the requirements of modern formal analysis with Aristotelian Logic (i.e. with a respectful interpretation of Aristotle's Logic). It presents Syllogistic as the exposition of the logic underlying the demonstrative method peculiar to science (as conceived by Aristotle), this logic being a natural deduction system in which first figure syllogisms are "applications of rules of inference" and those of other figures are "derived arguments." This deduction theory "is fundamental in the sense that it presupposes no other logic, not even propositional logic." In addition, it is "complete in the sense that every valid Pc^{23} argument composed of categorical sentences can be 'demonstrated' to be valid by means of a formal deduction in the system." Aristotelian natural deduction theory comprises, according to Corcoran, a language L consisting of the four logical constants A, E, I, O (renamed by the author as A, N, S, \$ after the English determinants all, none, some, some not) and an infinite set U of non-logical constants (the terms with no determinate content, symbolized by letters, which Aristotle uses). To language L corresponds a syntactic system S formed by significant terms (with determinate content) "correlated with the secondary substances²⁴ (sortal — or class — universals) [...]. The interpretation ix of the content word x is the extension of the secondary substance correlated with x." This interpretation "assigns to each content word a set of primary substances (individuals) which 'could be' the extension of a secondary substance" (Corcoran, 1974b, p. 98-104). As the above mentioned authors do, Corcoran presupposes an existential import for every term of the Aristotelian logical language: "i is an interpretation of L if and only if j is a function which assigns a non-empty set to each member of U." Corcoran adds: "This would explain the so-called existential import of A and N sentences. Notice that, according to this view, existential import is a result of the Semantics of the terms and has no connection whatever with the meaning of 'All'." (p. 104)25

Now, putting differences aside,²⁶ all these authors recognize the rigorousness and completeness of Aristotelian Logic, not feeling the need of propping it up on other supposedly underlying formalisms such as propositional calculus. But, above all, they make evident that Aristotelian formalization is not merely *calculus-oriented* but also *semantic*; it only intends to show the regularities underlying *natural language* in order to facilitate its use as a rigorous argumentation tool at the service of scientific inquiry. This inquiry requires, in its turn, that any explicative proposition be directly or indirectly based (through argumentation) upon indemonstrable previous principles that are the object of "first philosophy." Hence Aristotle's Logic, its formal strictness not being lessened, is inseparable from his Metaphysics.

For the same reason, since metaphysics is the knowledge of, or merely the inquiry about, the simplest elements of our representation of reality, it is unacceptable to circumscribe the Aristotelian logical analysis, as the aforementioned authors do, to the "higher" logical level which is that of syllogistic theory, or rules of proposition combination with deductive aims, without recognizing t hat this analysis presupposes the previous analytical level of simple statement. Therefore, it seems inexcusable to us, even if those authors' conclusions are globally accepted, to take a further step attempting a coherent formal interpretation of the Aristotelian theory of intrapropositional links between terms. In other words, not to accept those links as unanalyzable²⁷ "logical constants," but to decompose them, if possible, into simpler elements.

²³ "Premise-conclusion."

²⁴ In Aristotelian sense, cf. *Categories* 5.

²⁵ The remark coincides with the Kneales' interpretation mentioned above.

²⁶ Granger, for instance, would disagree with Corcoran in the appeal the latter makes to the notion of "set of *individuals*" as a semantic correlate of the terms; instead, he talks about "classes of *models* or *representations*," thus avoiding the merely extensional approach and getting closer, in our view, to the original Aristotelian stance.
²⁷ This is the "abstentionist" approach adopted first by Łukasiewicz and followed by (as far as we know) every subsequent scholar.

Is Syllogistic a combinatorial device dealing with propositions or rather with terms?

As a matter of fact, a rigorous interpretation of the Aristotelian theory of deductive reasoning, or syllogism, leads us to lay stress on the links between terms rather than between propositions. Indeed, this emphasis occurs in the syllogism's definition/description given by Aristotle in the first paragraphs of *Prior Analytics:*

A reasoning²⁸ is a statement²⁹ in which, certain things being stated, something other than what is stated follows of necessity from their being so. By *from their being so* I mean that it follows because of them, and by *what follows because of them* I mean that no further term is required from without in order to make the conclusion necessary (24b19-22).

And later on, when introducing the commonly named first figure:

Whenever three terms are so related to one another that the last is in the middle as a whole, and the middle is either in or not in the first as a whole, there must be a perfect reasoning between the extremes (25b31-35).

Furthermore, in many passages of the same work, instead of talking about propositions or premises (protáseis), istotle does so about intervals (diastémata), obviously between terms.³⁰

Now if syllogistic "calculus"³¹ is basically an "operation" carried out with several (at least three) terms, it becomes clear that the analysis must focus on the nature of the *operators* (Corcoran's "logical constants"). As we have seen, such analysis is the core object of *De Interpretatione*, whose definitions of those "operators" summarily reappear at 24a18-19:

By universal I mean what holds of all or none; by particular, what holds or not of some or not of all.

That one term should be in another as in a whole is the same as for the other to be predicated of all of the first. We say that it is predicated *of all* of another, whenever nothing can be found of which the other term cannot be asserted; to be predicated *of none* must be understood in the same way (24b27-35).³²

It seems clear, then, that the aforesaid operators are four possible *relationships* of attribution of a predicate to a subject. The term 'relationship' is neither arbitrary nor anachronic; the author of *Prior Analytics* himself uses the corresponding verbal form *échein prós* (cf. 25b32). Given the characterisation that Aristotle makes of

²⁸ Within the Aristotelian text, we translate *syllogismós* by "reasoning," for the *Organon's* author uses that term in a much wider sense than the one strictly corresponding to the deductive schemes studied here.

²⁹ In a broad sense (discourse) and not as a simple proposition (so it is understood by Łukasiewicz, who is conditioned by his prejudice of denying inferential character to the syllogism).

³⁰ Cf., for instance, 35a12, 31; 38a4; 42b29. The same occurs in Posterior Analytics.

³¹ In fact, the term *syllogismós* refers to a certain notion of "computation" of several factors in order to obtain a result. ³² Aristotle shows here, as the expression of a universal predication, the equivalence between the formula that presents the subject, extensionally speaking, as a sub-set of the predicate set and the formula that presents the predicate as applied to the whole subject. He also notes an equivalence that should make reflect all those who deny the existence of quantification in Aristotelian Logic — to predicate something about every part of a subject is the same as to deny the possibility of not applying the predicate to some part of it. In short, something like the equivalence between the formulæ (1) and (1 bis) above.

those relationships, we could symbolize³³ them in the following way, which is much more descriptive than the four traditional vowels (*P* designates the predicate and *S* designates the subject):

Universal affirmative:	P[S
Particular negative (negation of P[S):	P]S ³⁴
Particular affirmative:	P+S
Universal negative (negation of P+S):	P–S

The properties of these relationships obviously are:

[is transitive and asymmetric ³⁵
]	is neither transitive nor symmetrical
+,-	are symmetric but not transitive ³⁶

Together with the presentation of the terms in the standard (predicatesubject)³⁷ order proposed by Patzig, this symbolism allows to show quite clearly the deductive strength of the first figure syllogistic modes where the conclusive relationship between the extreme terms naturally appears as the *product* of each of their relationships with the middle term. Actually, in determining this product negative and particular relationships prevail over affirmative and universal, as is attested by the famous mnemonic hexameter of traditional Logic *peiorem sequitur semper conclusio partem:*

[[=[-[=- [+=+ -+=] []=]

The combinations made up of two negative or two particular (namely: - –, -], + +, +],]]) give a null product, as they do not grant any common semantic field shared by all three terms.

Applied to the four (perfect) modes of the first figure, and representing the relationships as *naturally* as possible (namely, linking the extremes with the middle term in a continuous string), this would give:

Barbara:	A[B[C=A[C
Celarent:	A-B[C=A-C
Darii:	A[B+C=A+C]
Ferio:	A-B+C=A]C

³³ As is well known, Aristotle does not symbolize them (he only symbolizes terms). This reinforces the idea that the linguistic level of the operators is a *metalinguistic* one in respect of that of the terms. Terms are *mentioned*, operators are used.

³⁴ This symbolization of the particular negative relation has the virtue of intuitively indicating its nature, opposed to that of the universal affirmative; it suggests, at once, one of the three possible semantic interpretations of this predicative relation, according to which the predicate's extension is included in the subject's extension (the other way round of its opposite).

³⁵ In theory, it would be possible to attribute the reflexive property to it, as Łukasiewicz does with this relation and with the particular affirmative, reformulating them as identity relationships. But such property lacks interest to Aristotle, at least in Syllogistic, and so (unlike the Polish logician, who needs it to complete the picture of his axiomatic interpretation of *Prior Analytics*) he never makes appeal to it.

³⁶ This symmetry would allow, without appealing to the *ékthesis* procedure used by Aristotle in Pr. *An.* I 2, an explanation of the convertibility of A–B and A+B. Very different, as will be shown below, is the case of "partial convertibility" of A[B into B+A, which at first sight does not follow from the properties of the relation. ³⁷ This is the order generally followed by Aristotle. E.g.: "If A is predicated of every B, and B of every C, A must be predicated of every C. [...] Similarly, if A is predicated of no B, and B of every C, it is necessary that A will belong to no C" (Pr. *An.*, 25b37-a2).

The modes of the other two figures were seen as imperfect by Aristotle due to their "lack of steps" to demonstrate in a direct way the relationship between the extreme terms. So they had to be converted, through a few transposition moves, into a mode of the first figure, or proved by *reduction ad impossibile*.

So far we have not said, on the whole, anything that goes beyond what the above mentioned authors maintain. But we must part company with them if instead of considering the four operators, logical relationships, or constants, as *external* to, or *superimposed* upon, the terms, and the terms themselves as *semantically indifferent* to the relationships the operators impose on them, we suppose that the form of the relationship somehow modifies the content of the terms.

But why should we suppose so? We feel obliged to do it for the following reasons:

The first reason has to do with the fact that, in contrast with what we pointed out at note 35 *supra*, the justification of the fundamental rules of total convertibility of the propositional schemes A–B and A+B, and that of partial convertibility of the A[B scheme (in B+A), has a *semantic* character, as is well shown by Granger (1970, p. 299f). And it does so because it derives from a *contraposition principle* (the semantic version of the *modus tollens* which excludes a void interpretation of the terms) based upon the so-called *ékthesis*. Aristotle explains:

Now if A belongs to no B, B will belong to no A: for if it does belong to some B, say, to C, it will not be true that A belongs to no B: for C is one of the Bs (Pr. An., 25a14-16).³⁸

This appeal to the *ékthesis* or *exposition* was implicitly criticised by Alexander of Aphrodisias (*in Pr. An.*, 32.32) as a move from the logical to the observational sphere and held by Łukasiewicz to be totally accessory and marginal (although valid from the logical standpoint if an existential quantifier is added to the proposition that has the "exposed" term as its subject). *Ékthesis* actually is, according to Aristotle, the cornerstone of the convertibility rules: firstly, the convertibility of A–B into B–A and vice versa (and, by the same token, that of A[B into B+A); secondly, on the basis of what we have just said, the convertibility of A+B into B+A (which leads, together with the convertibility of A[B into B+A, to the implication A[B \div A+B). Now, without these conversion rules and, to a lesser extent, without the aforementioned contraposition principle, the syllogistic modes of the first figure would be the only endowed with demonstrative power. Thereupon, little doubt can there be about the importance of the *ékthesis* procedure in Aristotle's Syllogistic.

The application of an existential quantification to the proposition resulting from *ékthesis* (as Łukasiewicz rightly proposes, although he does not admit that this formulation underlies Aristotle's own view of the matter) lets us wonder why it would not also be possible to apply the existential quantification to A+B type propositions.

The second reason refers to the dubious soundness of a supposition shared by all modern interpreters of Aristotelian Logic; namely, that Aristotle does not consider the possibility of void, referenceless terms. Were this assertion true, we would then have to inquire the meaning of passages such as the following:

[...] Socrates is well is contrary to Socrates is sick, yet not even with these is it necessary always for one to be true and the other false: for if Socrates exists one will be true and one false, but *if he does not* both will be false; indeed, neither Socrates is sick nor Socrates is well will be true if Socrates himself does not exist at all. [...] But with

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³⁸ We emphasize the words exactly corresponding to the argumentation through *ékthesis*.

an affirmation and negation one will always be false and the other true whether he exists or not; for if Socrates exists it is clear that either *Socrates is sick* or *Socrates is well* will be true or false, and equally if he does not: for if he does not exist he is sick is false but he is not sick true (Cat.,-13b14-34).³⁹

Our interpretation of this passage, based on the translation of Ackrill (1963, p. 37), as a proof of Aristotle's admission of empty terms (*Socrates*, in the occurrence) could only be refuted on the previous denegation of existential import to the verb *eînai* here and throughout the Aristotelian corpus. In this respect, we refer to Hintikka (1986), according to whose authoritative criterion the passage just quoted, among many others,⁴⁰ calls for an unequivocally existential interpretation of *eînai*.

Some could object that the restriction of the Semantics of terms to the point of excluding their emptiness is valid only for Syllogistic. However, it is well known that Syllogistic stands on the analysis of simple propositions that is carried out in *Int.*, which in turn is one of Aristotle's texts containing more allusions to referenceless terms. Besides that, Aristotle never warns us, neither in *Pr. An.* nor in *Posterior Analytics* (hereinafter *Po. An.*), about his widening or limiting the semantic scope of the terms he uses. In contrast, it is clear that in *Po. An.*, as Hintikka (1986, p. 89-92) points out, one of the main aims of scientific reasoning is to demonstrate the existence of a reference for certain terms (which obviously implies the possibility of their lacking it).

The third ground to go beyond an exclusively denotational Semantics in the interpretation of Aristotle's Logic lies in his analysis of a procedure traditionally called "obversion of equipollent propositions." This procedure consists of the passage from an affirmative proposition with a nominal predicate of a certain sign (e.g. "Every human being is not-white") to a negative proposition with a nominal predicate of the opposite sign (e.g. "No human being is white").⁴¹ Let's remark, by the way, that in a typical modern notation such as the first order predicate Logic this transformation would be irrelevant because one and the same formula corresponds to bothexpressions, namely $\forall x(Hx \rightarrow \sim Bx)$. Instead, from an Aristotelian standpoint, a transformation between logically and semantically heterogeneous structures has occurred. In other words, according to Aristotle, the above statements are not equipollent propositions.⁴² He says:

No man is just follows from Every man is not-just, while the opposite of this, Not everyman is not-just, follows from Some man is just, for there must be one (Int., 10, 20a20-23).⁴³

³⁹ Cp. Categories, 13b14-19, 27-33. Whoever could be tempted to object that this translation relies on a biased interpretation of the verb εἶναι should read without prejudice the Greek original of this passage from Categories: ὄντος μὲν γὰρ Σωκράτους ἔσται τὸ μὲν ἀληθὲς τὸ δὲ ψεῦδος, μὴ ὄντος δὲ ἀμφότερα ψευδῆ· οὕτε γὰρ τὸ νοσεῖν Σωκράτη οὕτε τὸ ὑγιαίνειν ἀληθὲς μὴ ὄντος ὅλως τοῦ Σωκράτους. [...] ἐπὶ δέ γε τῆς καταφάσεως καὶ τῆς ἀποφάσεως ἀεί, ἐἀν τε ἦ ἐἀν τε μὴ ἦ, τὸ μὲν ἕτερον ἔσται ψεῦδος τὸ δὲ ἕτερον ἀληθές. The idea of esse simpliciter or existence — in this case, its negation — is sometimes strengthened by the unequivocal adverb" ὅλως ("definitely", "absolutely") applied to the verb to be.

⁴⁰ E.g. *De interpretatione* 21a25-27; *Po. An.* II 1-2; *Sophistical Refutations* 167a4-6; as well as many passages in *Physics* VIII and *Metaphysics* XII.

⁴¹ After Aristotle, we call a "negative proposition" that in which the negation is directly connected to the verb or verbal phrase — 'is,' in the occurrence.

⁴² Except for those cases in which the subject-term is a singular term as in *Socrates is not wise* = *Socrates is not-wise* (cf. *De interpretatione,* 20a23-26).

⁴³ Bocheński (1956-1962), on the contrary, translates (erroneously) this passage as follows: "[The proposition] *No man is just* is followed by *Every man is not-just*. The contradictory of *Some man is just* is *Not every man is not-just*; for someone must be [just]" (original in German). Now, needless to compare it with the Greek original, it becomes evident that the contradictory of *Some man is just* cannot be at any rate *Not every man is not-just*, but *No man is just*, which in no way is equipollent of the previous one. Besides that, according to the Greek text, the arrangement of *Every man is not-just* and *No man is just* is exactly the opposite of the arrangement proposed in Bocheński's version.

And much more explicitly:

Now, in establishing or refuting, it makes some difference whether we suppose not to be this and to be not-this to have identical or different meanings, e.g.: not to be white and to be not-white. For they do not mean the same thing, nor is to be not-white the negation of to be white, but rather not to be white. The reason for this is as follows: The relation of he can walk to he can not-walk is similar to the relation of it is white to it is not-white; so is that of he knows what is good to he knows what is not-good. [...] If then not able to walk means the same as able not to walk or as not to walk, these will belong at the same time to the same thing (for the same thing can both walk and not-walk, and knows what is good and what is not-good), but an affirmation and a negation which are opposed to one another do not belong at the same time to the same thing. As then not to know what is good is not the same as to know what is not-good, so to be not-good is not the same as not to be good. [...] Nor is to be not-equal the same as not to be equal: for there is something underlying what is not-equal, and this is the unequal, but there is nothing underlying the other. That is why not everything is either equal or unequal, but everything is equal or is not equal. Further it is a not-white log and it is not a white log do not belong to something at the same time. For if it is a not-white log, it must be a log: but that which is not a white log need not be a log at all (Pr. An., 51b5-31).

It is clear, then, that we are dealing with pairs of propositions which differ in nature so that one member of the pair can be inferred from the other *but not vice versa;* otherwise, the case would be a double implication relationship, i.e. one of equivalence. Now the sense of the simple implication is, according to Aristotle, just one that goes from the affirmative to the supposedly equipollent negative, *but not the other way round.*⁴⁴ Indeed, besides the first of the texts just quoted, we have (at 19b26-35):

"Let us clear what we meant from the following diagram:

<a>	
A man is just	its negation being: A man is not just;
<d></d>	<c></c>
A man is not not-just	(negation of) A man is not-just ⁴⁵

Certainly, 'is' and is not are here added to just and to not-just. This is how these are arranged, as is said in the Analytics [51b36-52a17]. They behave in a similar way if the affirmation is about the name taken universally, e.g.:

<a>	$\langle B \rangle$
Every man is just	Not every man is just
<d></d>	<c></c>
Not every man is not-just	Every man is not-just"

And above all:

The relation of these to one another is as follows. Let A stand for *to be good*, B stand for *not to be good*, let C stand for *to be not-good* and be placed under B, and let D stand for *not to be not-good* and be placed under A. Then either A or B will belong to everything, but they will never belong to the same thing. And B must belong to everything to which C belongs (for if it is true to say *it is not-white*, it is also true to say *it is not white*: for it is

⁴⁴ With the exception mentioned at note 42 supra.

⁴⁵ As noted above (see note 12 *supra*) and will immediately be proved, Aristotle states the propositions of this and other diagrams (the well-known "opposition quadrilaterals") in a clockwise order. That is why the proposition placed at the lower right angle precedes in the text the one placed on its left.

impossible that a thing should simultaneously be white and be not-white, or be a notwhite log and be a white log; consequently, if the affirmation does not belong, the denial must belong.); but C does not always belong to B (for what is not a log at all cannot be a not-white log either). In contrast, D belongs to everything to which A belongs (for either C or D belongs to everything: but since a thing cannot be simultaneously white and not-white, D must belong to everything to which A belongs); for of that which is white it is true to say that it is not not-white; but A is not true of every B (for of that which is not a log it is not true to say A, viz. that it is a white log; consequently D is true, but A is not true, i.e. that it is a white log). It is clear also that A and C cannot together belong to the same thing, and that B and D may belong to the same thing" (*Pr. An.,* 51b36-a14).⁴⁶

Hence we would have the following scheme:

A	В
To be good	Not to be good
D	С
Not to be not-good	To be not-good

In this scheme Aristotle explicitly asserts that, besides the obvious A + B and D + C (contradiction) relationships, A + C (contrariness) and B - D (subcontrariness) as well as $C \rightarrow B$ and $A \rightarrow D$ (implication) occur, but the converse of the last two do not. The very same relationships, arranged in the very same order, appear likewise here and in the classical opposition quadrilateral.

It is crystal clear, then, that affirmative statements necessarily entail the corresponding negative ones, the latter having a nominal predicate which stands as contradictory of the predicates of the former. This entailment does not obtain the other way round. Aristotle gives the reason thereof:

For if it is true to say *it is not–white*, it is also true to say *it is not white*: [...] but [...] what is not a log at all cannot be a not–white log either. [...] of that which is white it is true to say that it is not not–white; but of that which is not a log it is not true to say *A*, viz. that it is a white log.

What could this mean but that in a negative statement it is possible to interpret the subject-term as an *empty term*? In such a case it is evident that from this proposition it is not possible to infer another one in which the same subject-term be endowed with a content; i.e. from the non-existence of something it is impossible to infer anything entailing its existence.

A lot has been written on the extent to which the notion of existence was explicitly present in the ancient Greek verb *eînai* (in *Pr. An., hypárchein*) and its relatives. Probably nothing can be added to Hintikka's (1986) and, above all, to Kahn's (2003, 1986) conclusions. According to them, the existential import of the verb *eînai* is implicit in its *copulative* (or, in Russell's words, *predicative*)⁴⁷ use and,

⁴⁶ In contrast with the previously quoted passages, in which the letters assigned to the propositions were mere additions of our own, in this case it is Aristotle himself who appeals to the expedient. This is the first known text in which a symbol is assigned not to a term but to a whole statement. We see, thus, that the Stoic development of the symbolic Logic of statements already had some basis in Aristotle.

⁴⁷ Hintikka emphatically asserts that the verb *eînai* cannot be dissected into the well-known quadripartite semantic distribution proposed by Frege and Russell, by means of which those authors attempted to dispel the ambiguity between an is: (a) identity sign, (b) predicate of existence, (c) predicative copula, and (d) inclusion sign. Kahn holds that to all those meanings or uses (we would rather say "values," for we do not find it possible to separate meaning from use) a veridical value (*is* = *it is true that*) must be added. Kahn holds also that all of them are closely related to each other (as if all of them had a "common root") in the ordinary use of *eînai*.

therefore, it never constitutes a predicate in the strict sense as it started to be in the philosophical jargon from the famous Anselmian (or "ontological") argument onwards. In Aristotle's case, we could say that the existential value of *eînai* appears perfectly explicit when it stands as the only predicative element together with the subject (what in the Scholastic tradition was called esse ut secundum adjacens); but the "structure underlying" both that use and the standard copulative use (esse ut tertium adjacens) corresponds with striking parallelism to the modern logical formulæ in which the variable is bound through an "existential" quantifier. Actually, the examples in the passage just guoted from Pr. An. clearly suggest a reformulation of (1) the statement "The log is white" in "There is something that is a log and is white." Analogously, (2) "The log is not-white" can be rewritten as "There is something that is a log and is not white." In contrast, (3) "The log is not white" could be thus rewritten according to Aristotle, "There is something that is a log and is not white, or there is something that is not a log at all and, therefore, is not a white log either."48 It is obvious that in a language like ours nobody would understand the statement (3), as we reformulate it, in the sense of the second member of the disjunction. That is why we can only assume that for a Greek from Aristotle's times statement (3), regardless of its formal structure of subject-copula-predicate, meant something similar to our expression (4) "There is not a white log."49

On the above grounds it seems necessary to propose a new formulation of the elementary propositions, as interpreted by Aristotle, in terms of first order predicate Logic. In this formulation, the referential scope of the terms varies according to the type of relationship established between subject and predicate. This allows us to solve all the difficulties encountered when we first approached the matter by means of modern quantification theory.

If 'is' (ἔστι) connotes existence, 'is not' (οὐκ ἔστι) connotes non-existence

Actually, the formulation we propose here goes as far as to bring to its last conclusions the criteria constantly followed by Aristotle in order to determine the relationships between propositions that share identical significant terms while differing as affirmation and negation; namely, to determine for each proposition what its contradictory, i.e. its bare negation, is.

If, then, as it seems compulsory to do in order to account for the existential import of the copula, we begin by modifying $\forall x(Hx \rightarrow Bx)$ (the universal affirmative) through the adjunction of an existential quantification of the variable, we obtain:

⁴⁸ That is to say (although Aristotle obviously does not state it in such terms) the proposition has a *disjunctive* semantic structure. The subject has a referent and the predicate does not apply to it, or it simply lacks any reference and applying the predicate to it is nonsensical.

⁴⁹ In this regard, Kahn (1986) quotes an illuminating proposal, by then still unpublished, by Mohan Matthen. This author holds that in constructions with *eînai* you have a "predicative complex," defined as "an entity formed from a universal and a particular *when that particular instantiates the universal.*" Kahn comments: "In grammatical terms, a predicative complex (or rather, its linguistic expression) is the attributive transform of an ordinary copula sentence: corresponding to *X is Y* we may assume the existence of a logically equivalent predicative complex, *the YX exists*. Thus, for (1) *Socrates is healthy* we have the corresponding (2) *The healthy Socrates exists*, where the truth conditions of (1) and (2) are assumed to be identical. Furthermore, truth conditions will also be the same for the veridical transform of (1), namely (1A) *It is the case that Socrates is healthy.*" According to Kahn, this interpretation would easily show "why our conventional dichotomy between existence and copula imposes a choice upon the interpreter that corresponds to nothing in the Greek data (Kahn, 1986, p. 27, n. 46).

(9) $\forall x(Hx \rightarrow Bx) \cdot \exists x(Hx)$, or its equivalent (5) $\sim \exists x(Hx \cdot \sim Bx) \cdot \exists x(Hx)$

It is obvious that denying that formula to obtain its contradictory opposite (the particular negative) will result in the already known formula:

(7) $\exists x(Hx \cdot \sim Bx)v \sim \exists x(Hx)$

Similarly, if we start from the particular affirmative such as stated above, (3) $\exists x(Hx \cdot Bx)$, its negation or contradictory opposite (the universal negative) will be expressed in the formula (2 bis) $\sim \exists x(Hx \cdot Bx)$, and not with the extremely inconvenient (6) $\sim \exists x(Hx \cdot Bx) \cdot \exists x(Hx)$.⁵⁰

Now, by virtue of De Morgan's second law, (2 bis) is equivalent to:

(10) ∀x(~Hxv~Bx)

The truth condition of (10) obviously lies on the interpretation of at least one of the terms, or of their intersection, as empty. Similarly, formula (7) proves itself true whether both do not overlap, if the predicate does not cover the whole extension of the subject, or when the subject-term is void.

Now it is trivial to see that being:

A: ~∃x(Hx·~Bx)·∃x(Hx)	O: ∃x(Hx·~Bx)v~∃x(Hx)
I: ∃x(Hx·Bx)	<i>E</i> : ~∃x(Hx·Bx)

the Aristotelian truth-functional tables, as shown at the beginning of this paper, hold perfectly. Therefore, our interpretation perfectly accounts for all the properties assigned by Aristotle to the relationships between quantified propositions. These relationships obviously are truth functions, despite that (i) the *Organon* does not develop a Logic of propositions *stricto sensu* and (ii) the proof of their validity compels to transcend the consideration of the propositions as atomic units and to decompose them into their elements, both *categorematic* (the terms)⁵¹ and *syncategorematic* (propositional functors, quantifiers, and negations). But we realize, above all, that the assumption that grounds such an interpretation (assumption which is firmly supported by texts of *Categories, Int.,* and *Pr. An.*) considerably "modernizes" the archaic denotative Semantics that has been so stubbornly attributed to the founding father of Logic by almost every modern Aristotelian scholar. It does so by ascribing to Aristotle the acknowledgement of "null classes"⁵² or referenceless terms.

Nonetheless, it must be kept in mind that our modern formal transcription of the Aristotelian formalisms has several restrictions that prevent us from talking about a "transformation without rest" of the Aristotelian language into that of mathematical Logic or vice versa. One of the most impressive evidences thereof is the already analyzed "asymmetric obversion" that forbids, in transcribing propositions that involve a negation, writing \sim Ax in order to represent both x is not A and x

⁵⁰ This is the key element in our interpretation: the non-adjunction of the existential quantification \$x(Hx) to the standard formula of E. Among others, the passages quoted at the end of the previous section allow us to do so. It is evident there the admission by Aristotle of void terms in propositions with negative verb ("verb" being the copula or assertive functor, as distinguished from the predicate itself, that can be a name, an adjective, or a not merely functional verb with semantic content of its own.

⁵¹ The fact that Syllogistic is a calculus operating with terms rather than with propositions upholds the strategy of transcending mere propositional Logic.

⁵² The only exception here would be that Aristotle does not seem to consider the possibility of taking the null class as a sub-class of every other class.

is not-A. (Paradoxically, this formal "anomaly" puts us on the track leading to the admission in Aristotle of a Semantics strikingly close to that of modern logical languages.) But the most important restriction is no doubt the one that compels to interpret *A*, as a matter of fact, as the conjunction of *A* and *I*. Such an interpretation deprives the universal quantifier of its genuine *universal* nature because universal quantification points not only to what is *actually* predicated of the variable but also to what is *possible* to predicate of it, and must, therefore, have a *hypothetical*, not a *categorical*, semantic interpretation.

Anyway, once we have reached this point, we should weigh the implications of our interpretation for the Aristotelian ontology, as every semantic theory entails an ontological theory, i.e. a theory of the links between the structure of language and the structure of reality.

"Being is not the essence of anything"53

Every affirmative proposition (not only those containing a copula verb like to be plus a nominal predicate, but also propositions with predicative verb) somehow connotes the existence of what is denoted by the subject-term (and, by the same token, of what is denoted by the predicate). Although the opposite could seem to be the case, the reason for this is that in ancient Greek there is no verb unequivocally referring to our notion of existence. The verbs $\varepsilon iv\alpha_1$ and $\delta \pi \alpha \rho \chi \varepsilon iv$ appear as the most apt to play this role of existence predicates; yet in the Organon they operate as mere connectors of terms, lacking a semantic content of their own. As Aristotle explicitly says:

For not even to be or not to be is a sign of the actual thing (nor if you say simply that which is); for by itself it is nothing, but it additionally signifies some combination, which cannot be thought of without the components (*Int.* 16b21-25) (Ackrill, 1963, p. 45).

Eînai with a "zero" nominal predicate (secundum adiacens) clearly seems to have a meaning that can be translated in expressions such as "exists" or "there is." Nevertheless, Aristotle carefully warns "it is not a sign of the actual thing," not even when used in its participial substantivized form which is the Greek idiom canonical for referring to what is *real*. In return, what is typical of *eînai* is that it "additionally signifies some combination [$\pi \rho o \sigma \sigma \eta \mu \alpha i \nu \epsilon_1 \sigma \delta \nu \eta \epsilon_2$."

Now, as Aristotle puts it at 16b6-7, "a verb is what additionally signifies time [...] and it is a sign of things said of something else" (Ackrill, 1963, p. 44). These are, then, the essential features of every verb, including "to be": The verb establishes a link (*sýnthesín tina*) through which the meaning of a term (predicate) is included in the meaning of another term (subject), within the limits imposed by the reference to a certain instant or period within the temporal flux. Leaving this crucial feature aside, "a verb is a name and signifies something [...], but it does not yet signify whether it is [the case] or not" (*Int.* 16b19-21; Ackrill, 1963, p. 45). So it is not the verb as an independent term, but the *temporally determined predicative synthesis* that bestows on the statement the extralinguistic reference that we call "existential import."

The meaning of *eînai* comes closer than any other verb to the notion of existence (to the point of not merely connoting but openly denoting it when in position of *secundum adiacens*) because it is the most "meaningless" verb, a verb with minimal semantic content of its own (hence its overwhelmingly dominant use in the

⁵³ τὸ δ' εἶναι οὐκ οὐσία οὐδενί (Po. An., 92b13-14).

position of the copula *par excellence*, a mere connector between significant terms).⁵⁴ In other words, the outstanding capability of *eînai* to express the existence (or, in the negative form, the non-existence) of something follows from its being a purely grammatical element with no meaning of its own but, at the same time, crucial for making statements endowed with truth-value.⁵⁵ But if existence is expressed by a term that does not signify but only *additionally signifies*, we must conclude that, in Aristotle's Greek, existence is not a meaning proper but only an *additional meaning*. Therefore, regardless of its being justified or not in general terms, the Kantian *dictum* "existence is not a predicate" is fully justified concerning Aristotle.

This conclusion is strengthened by the arguments offered by Kahn (1986) in order to demonstrate that the existential use of *eînai* derives from, and is connected with, its *veridical* use (which, unlike the existential use, was recognized as such by Aristotle in *Metaphysics* Δ 7).⁵⁶ This veridical use is closely associated with the predicative or copulative use. The following passage is especially eloquent in this regard:

An affirmation [κατάφασις] is a statement asserting something as united with something [τινὸς κατὰ τινός], and a negation [ἀπόφασις] is a statement asserting something as separated from something [τινὸς ἀπὸ τινός]. Now it is possible to state of what does hold that it does not hold, of what does not hold that it does hold, of what does not hold that it does hold. Similarly for times outside the present. So it must be possible to deny whatever anyone has affirmed, and to affirm whatever anyone has denied (*Int.* 17a25-32).⁵⁷

In statements including both a verbal predicate or a copula working as a *tertium adiacens*, the existence of what the proposition terms designate is *additionally signified* by the affirmative copula (represented by the verb *eînai* or by the morphemes of time, person, number and aspect of categorematic verbs) and its (absolute or relative) non-existence is likewise *additionally signified* by the negative copula. But, what is the case when the verb *eînai* is used as a *secundum adiacens*, i.e. when it is apparently used as a predicate of its own?

Following a suggestion made by Owen (1960, 1965), Dancy (1986) holds that a proposition such as *Homer is* is but a simplified version of *Homer is a human being*; in other words, it is but the *ellipsis* of a predicative proposition in which the (absent) predicate expresses the essence of what is referred to by the subject.

The trouble with this interpretation is that it frontally collides with a distinction clearly established by Aristotle in *Po. An.* (II 1-2) between the inquiry *whether something is* ($\epsilon i \ \delta \sigma \pi$: its existence) and the investigation of *what is it* ($\pi i \ \delta \sigma \pi$: its essence). This is why we concede much more credibility to another interpretation that brings the structure of the aforementioned statements near the existentially quantified schemes of contemporary Logic; thus, in the above example, *Homer is* = *something is Homer* = $\exists x(x=Homer)$. *Homer,* so construed, ceases to be (logically) a subject and becomes a predicate. The same holds for the negation *Homer is* not = nothing is *Homer* = $\sim \exists x(x=Homer)$.

⁵⁶ See note 47 supra.

⁵⁴ As Peter Abelard wittily notes in his *Logica ingredientibus*, the copula "copulat tantum et non copulatur" (ed. Geyer 351) — it combines subject and predicate without "mingling" with them.

⁵⁵ Aristotle calls them *apophantic* (*assertive* or *declarative*, in our grammatical terminology). He characterizes them as "those in which there is truth or falsity" (*Int.* 17a2-3) or, more precisely, "The simple statement [$\dot{\alpha}$ πόφανσις] is a significant spoken sound about whether something does or does not hold [$\dot{\upsilon}$ πάρχει] in one of the divisions of time)" (*Int.* 17a22-23).

⁵⁷ The idea of "union," associated with the affirmative assertion, is strengthened by its contrast with the idea of "separation" (conveyed by the preposition $\dot{\alpha}\pi\dot{\alpha}$) associated with the negation. In order to emphasize that feature, our reading of the first two sentences quoted departs significantly from Ackrill's version.

This device is the same as above in the treatment of the standard copulapredicative statements, e.g. a man is white = something is a man and is white = $\exists x(Hx \cdot Bx)$, and every man is white = something is a man and everything that is a man is white = $\exists x(Hx) \cdot \forall x(Hx \rightarrow Bx)$, & c.

This prompts us to establish a thorough connection between Aristotle's (implicit) notion of existence and his notion of *subject* ($\dot{\upsilon}\pi\sigma\kappa\epsiloni\mu\epsilon\nu\sigma\nu$) — whatever has a substrate does exist. To exist is *to subsist*.

Now, according to Aristotle, *matter* ($\eth\lambda\eta$) is the absolute subject, that which can never become a predicate of anything. But as a pure subject matter cannot be described, or assigned any semantic content. It is only by virtue of the predicative function of the copula that it can leave the darkness of pure indeterminacy and go beyond its anonymous condition, acquiring a certain "identity" in the discourse.

Here we have another (not formal but strictly metaphysical) reason for considering the quantified formulæ of predicate Logic as the best symbolic⁵⁸ approach to the Logic of Organon. In fact, the so called "individual" variables, inasmuch as they represent the "pure" subject of the proposition, almost exactly correspond to Aristotelian *matter* (this concept being the theoretical result of a semantic-grammatical analysis of the propositions describing change processes, analysis carried out by Aristotle mainly in *Physics*, 1 5-9).

The formula $\exists x(x=Homer)$ itself (*vid. supra*) is a literal example of what the so-called "individuation principle" could be to Aristotle, the strict identification of a minimal subject (matter) with its maximal predicate (form).⁵⁹ As a matter of fact, in contrast with the normal attribution relationship in which the subject is partially determined by the predicate (determination which is partial even in the case of essential predication *there is such an x that x is a man*, for instance), what we have here is an exhaustive determination in which what is really attributed is the implicit intersection of all the possible predicates of the subject (predicates whose series, in each particular context, must be held to be finite if the term summarizing them has to be non-ambiguous).

In fact, therefore, to state the existence of an individual is a particular case of use of the verb "to be" as an *identity* sign, in a double sense. In the sense, first, of asserting the identity of an indeterminate "material" element with a set of determinations condensed in a singular term (which can be a proper name or a definite description) and in the sense, also, of "identifying" such indeterminate material element by means of its determination.

The "deep structure" of the "pure" existential propositions can be obscured when the apparent (grammatical) subject is a proper name (e.g. $\Sigma \omega \kappa \rho \dot{\alpha} \tau \eta \varsigma \dot{\epsilon} \sigma \tau i \nu$).⁶⁰ But it makes itself evident in cases such as $\dot{\epsilon} \sigma \tau i \tau \iota \varsigma \ddot{\alpha} \nu \theta \rho \omega \pi \sigma \varsigma$, in which the real, ultimate (and logical) subject, the indefinite matter, is indicated through the indefinite pronoun $\tau \iota \varsigma$. For that reason, "pure" existential propositions evenly admit translations like "a man exists" or "something is a man." That very same formula, substantivized without a verb ($\dot{\delta} \tau \iota \varsigma \ddot{\alpha} \nu \theta \rho \omega \pi \sigma \varsigma$), is used by Aristotle in exemplifying

⁵⁸ Always keeping in mind, however, that the modern formalisms constitute just an approximation, never a literal translation, in respect of Aristotelian logical-semantic system.

⁵⁹ The reduction, proposed by Aquinas, of the individuation principle to the *materia signata quantitate* (what amounts to consider the *quantity* as the only predicate determining individuality), represents an inadmissible degradation of the discernibility criteria (stated later on in the Leibnizian *identity of indiscernibles* principle). It lowers them to the "clonal" level of mere spatial criteria (those which allow to distinguish, for example, one scrap from another in a piece of cloth).

⁶⁰ This happens to Frege, who does not admit the unfolding of singular terms into formulæ containing an existential quantifier.

the $\pi\rho\omega\tau\eta \operatorname{ov}\sigma(\alpha)$, the *primary entity*, which he defines as pure subject. While a whole definite series of essential attributes as well as an indefinite series of accidental attributes (maximal comprehension)⁶¹ can be predicated of it, the primary entity cannot be predicated of anything (minimal extension). Indeed, the $\pi\rho\omega\tau\eta \operatorname{ov}\sigma(\alpha)$ cannot be predicated of anything, in spite of its being expressed by one or more predicates, for such predicate/s has/have contracted its/their extension and widened its/their comprehension up to the point of its/their exact coincidence with the subject (matter) it/they inform/s, so constituting a perfectly univocal and unique complex of semantic features.

The shift we propose, from a *predicative* notion of existence⁶² to a *subject-like* notion of it, means to think of existence as an implicit substrate, as a feature that "is assumed" to be rooted in the subject of every proposition whenever it be not explicitly denied.⁶³ It calls for the necessary identification of *existence* and *matter*. Were we right, this would easily explain Aristotle's belief in the eternity of the universe and, *a fortiori*, of matter (for how could existence cease to exist?),⁶⁴ as well as his being apparently "tempted" in *Metaphysics* Z 3 to identify $\eth\lambda\eta$ and $o\upsilon\sigmai\alpha$, temptation he can only resist by bestowing absolute ontological primacy to the $\pi\rho\omega\tau\eta$ o $\upsilon\sigmai\alpha$, construed as a whole ($\sigma\upsilonvo\lambda ov$) of matter and form.

To Aristotle, then, to on (being) as such, represented by the copula which gives predicative form (contingent or necessary) to the formless substrate of mere fact (existence), has no reality outside those very predicative structures (forms) it *asserts* but which are always, in their turn, either partial, transient, or both.⁶⁵

What we call *being*, verbalized in Greek as the apophantic operator or copula $\check{\epsilon}\sigma\tau\tau$ (and derivatively as the participle – $\check{\sigma}v$), works *apophantically*⁶⁶ as light does. It makes visible,⁶⁷ through the *essential* transparency of the predicate, the *existential* opacity of the subject and it *exposes* what is *supposed* without *posing* it either as subject or as predicate, transcending by itself the opposition between *essence* and *existence*.

Unlike the Parmenidean, Aquinian, or Heideggerian *being*, closed upon itself and pregnant of essential attributes or phenomenological-existential *pathos*, the Aristotelian *being* (mere link without content of its own) has in itself no entity. This feature enables it to identify itself with every concrete entity, to take on the perfection of every form as well as the precarious condition of any accidental

reality of the universe.

⁶¹ See, among other passages: Categories 2a13, 16, 22-25 & c.

⁶² Invention of a certain medieval philosophy, developed thereafter by mainstream modern philosophy, which was forced *eo ipso* to bring the object of metaphysics from the domain of *real being* into the realm of what is merely *possible*.

⁶³ In which case it simply happens that the pure material subject, the mere *something*, is deprived of the predicate that was embodied in it, as we saw at 51b28-32 & c., while the material subject itself remains "untouched."
⁶⁴ The "totalitarianism" of the Aristotelian conception of matter becomes apparent in the reduction of what is possible to what is materially possible (i.e. to the potentialities lying in the matter). Aristotle does not think at all, as modern philosophy (since Leibniz) does, of a plurality of possible worlds out of which the real world would be just a particular case. On the contrary, the only thing that deserves to be qualified as possible, for him, is what can come-to-be out of the matter underlying everything existing in the physical universe. Symmetrically, what always is (in a certain way) is, according to Aristotle, necessary. We can infer from there that Aristotle professes a particular form of the "principle of plenitude": Given the double framework of an infinite time and a finite space, everything that can come-to-be soner or later will come-to-be. Similarly, what is not necessary or what can cease to be, some time, sooner or later, will cease to be. On the contrary, what never ceases to be, what always is, is necessary. These considerations, stated at the end of De Cælo I, reflect a restrictive conception of the Semantics of modal statements, analogous to the existential restriction of the affirmative categorical propositions. A detailed study on the subject can be read in Waterlow (1982).

⁶⁶ ἀπόφανσις means, etymologically, "manifestation of a (previously hidden) thing."

⁶⁷ Hence its veridical function, from $å\lambda$ ήθεια — truth as a disclosure.

attribute, rooting both forms and accidents in the solid existential substrate of the subject. In short, by being nothing, Aristotelian *being* can be everything; although it can only be expressed inasmuch as it is "determined" by means of the different types of predicates (*categories*). By itself, without categorially "reducing" itself, it does not determine anything. It is neither *genus* nor *essence* of anything.⁶⁸ In return, the essence (oùơia) is the being of everything: $\tau \delta \tau i \tilde{\eta} v \epsilon ivai$.

To exist, therefore, is not only to subsist as pure subject but also to be determined by means of a predicate. To exist is to acquire a certain essence and certain attributes. Hence, Dancy and Owen's equivocal interpretation of existential statements as ellipses of essence statements. Hence, also, the conflation in the Greek word $o\dot{v}\sigma\dot{\alpha}$ of two different semantic contents: the subject-like one (*substance*) and the predicamental one (*essence*).

On the basis of the Aristotelian concept of $t \partial \partial n$, it becomes clear why the famous ontological argument about God's existence was radically flawed; for existence does not determine any subject, existence is itself the subject of every determination.

Now we can get a better understanding of the nature of negative propositions according to Aristotle. Rather than saying that those propositions cut off the connection between subject and predicate, cancelling at the same time the reference of either one or both terms, we must say that they cut out the possibility that the existence-subsistence of an absolutely undefined subject be determined through the predicate and be recognized as really existent.

Let us conclude, then, by stressing how the Aristotelian *ón*, in oscillating between the absolute existential indeterminacy of matter and the perfect categorical definability of essence, has thereby a flexibility that in exchange for all the equivocations it can entail for a strictly logical analysis of its role in the discourse protects it from many of the sophisms conveyed by certain formally more elaborated ontologies, whose very sophistication makes it easier for them to confuse their own conceptual constructions with reality.

References

ACKRILL, J.L. 1963. (ed.) Aristotle's Categories and De interpretatione. Oxford, Clarendon, 162 p.

ARISTOTLE. 1984. Organon. *In:* J. BARNES (ed.), *The Complete Works of Aristotle.* Princeton, University Press, 1256 p.

ARISTÓTELES. 1988. Tratados de Lógica II. Madrid, Gredos, 460 p.

BOCHEńSKI, I.M. 1956-1962. Formale Logik. Freiburg/München, Karl Alber, 648 p.

CORCORAN, J. 1974a. Aristotelian Syllogisms: Valid Arguments or True Generalized Conditionals? *Mind*, 83, p. 278-281.

CORCORAN, J. 1974b. Aristotle's Natural Deduction System. *In*: J. CORCORAN (ed.), *Ancient Logic and its Modern Interpretations*. Dordrecht, Reidel, p. 85-131.

DANCY, R.M. 1986. Aristotle and Existence. In: S. KNUTTILA; J. HINTIKKA (coords.), The Logic of Being. Dordrecht, Reidel, p. 49-80.

GRANGER, G.G. 1970. Le syllogisme catégorique d'Aristote. *L'Age de la Science,* III(4):281-310.

HINTIKKA, J. 1986. The Varieties of Being in Aristotle. In: S. KNUTTILA; J. HINTIKKA (eds.), The Logic of Being. Dordrecht, Reidel, p. 81-114.

KAHN, C. H. 1986. Retrospect on the Verb 'To be' and the Concept of Being. In: S. KNUTTILA; J. HINTIKKA (eds.), The Logic of Being. Dordrecht, Reidel, p. 1-28.
KAHN, C.H. 2003. The Verb 'Be' in Ancient Greek. Indianapolis, Hackett, 486 p.

KNEALE, W.; KNEALE, M. 1962. *The Development of Logic*. Oxford, Clarendon, 761 p.

⁶⁸ See Metaphysics 998b22ff, 1001a5f, 1040b18, 1045b3-7, 1052b23, 1059b27-33. Also Po. An. 92b14.

ŁUSAKIEWICZ, J. 1951-1957. Aristotle's Syllogistic from the Standpoint of Modern Formal Logic. Oxford, Clarendon, 222 p.

MENNE, A.; ÖFFENBERGER, N. (eds.). 1995. Über den Folgerungsbegriff in der Aristotelischen Logik. Hildesheim, Olms, 220 p.

OWEN, G.E.L. 1960. Logic and Metaphysics in Some Earlier Works of Aristotle. *In*: I. DÜRING; G.E.L. OWEN (eds.), *Aristotle and Plato in the Mid-Fourth Century.* Göteburg, Studia Græca et Latina Gothoburgensia XI, p. 163-190.

OWEN, G.E.L. 1965. Aristotle on the Snares of Ontology. *In*: R. BAMBROUGH (ed.), *New Essays on Plato and Aristotle*. London, Routledge & Kegan Paul, p. 69-98.

PATZIG, G. 1969. Die Aristotelische Syllogistik. Göttingen, Vandenhoeck & Ruprecht, 217 p.

PRIOR, A.N. 1962. Formal Logic. Oxford, Clarendon, ix + 329 p.

QUINE, W.V.O. 1982. *Methods of Logic.* New York, Henry Holt & Co., x + 334 p.

ROSE, L.E. 1968. Aristotle's Syllogistic. Springfield, Charles Thomas, vii + 149 p.

RUSSELL, B. 1956. Logic and Knowledge. London, George Allen & Unwin, xi + 382 p.

SAINATI, V. 1968. Storia dell'Organon aristotelico, I: Dai "Topici" al "De Interpretatione". Florence, Le Monnier, p. 5-26.

SMILEY, T. 1973. What is a Syllogism?. *Journal of Philosophical Logic*, 2, p. 136-154 STRAWSON, P.F. 1971. *Introduction to Logical Theory*. London, Methuen, 211 p.

WATERLOW, S. 1982. Passage and Possibility: A Study of Aristotle's Modal Concepts. Oxford, Clarendon, 165 p.

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