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Do multifactor models contribute to estimate the cost of equity capital in Brazil?

Os modelos multifatoriais contribuem para estimar o custo do capital próprio no Brasil?

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Abstract: This study investigates the contribution of well-known multifactor asset pricing models to estimate the cost of equity capital of Brazilian listed companies with the Capital Asset Pricing Model (CAPM), the three-factor model of Fama and French (1993), the Carhart (1997) four-factor model and a five-factor model that consists of an additional illiquidity risk factor. The sample are the returns of individual stocks comprising a portfolio of companies in the IBrX 100 stock index from July 2008 to June 2018. Distributions of individual company cost of equity capital estimates obtained with each model were compared among themselves in the full sample period and two sub-periods. The results suggest that adding extra risk factors to the CAPM does not always translate into different cost of equity capital estimates and significantly greater explanatory power. The practical implication is that the CAPM estimates may often be the same as those obtained by means of more complex models with the added bonus of the CAPM's simplicity.
Keywords – cost of equity capital, Brazilian market, multifactor models, portfolios of stocks.

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Resumo: Este estudo investiga a contribuição dos modelos multifatoriais de precificação de ativos para estimar o custo de capital próprio de empresas brasileiras de capital aberto como o Capital Asset Pricing Model (CAPM), o modelo de três fatores de Fama e French (1993), o modelo de quatro fatores de Carhart (1997) e um modelo de cinco fatores que consiste em um fator de risco adicional de iliquidez. A amostra é constituída dos retornos de ações individuais que compõem uma carteira de empresas do índice de ações IBrX 100 de julho de 2008 a junho de 2018. As distribuições das estimativas individuais de custo de capital próprio obtidas com cada modelo foram comparadas entre si no período amostral completo e em dois subperíodos. Os resultados sugerem que a adição de fatores de risco extras ao CAPM nem sempre se traduzem em estimativas de custo de capital próprio diferentes e poder explicativo significativamente maior. A implicação prática é que as estimativas do CAPM podem muitas vezes ser as mesmas obtidas por meio de modelos mais complexos com o bônus da simplicidade do CAPM.
Palavras-chave – custo de capital próprio, mercado brasileiro, modelos multifatoriais, carteiras de ações.

Introduction

Investors need the cost of equity capital to price securities and corporate managers require it to make better decisions about investment projects and their funding. Fama and French (2004) assert that the Capital Asset Pricing Model (CAPM) is the most widely used method for estimating the cost of equity capital of a given security and that it has been empirically tested for decades. These authors mention Black, Jensen and Scholes (1972), who rejected the traditional form of the CAPM but found a positive relationship between average returns and beta, and Fama and Macbeth (1973), who suggested that there was a positive relationship between risk and return, as well as Fama and French (1992), who argued that the fitted regression slope of historical returns on beta risk was rather flat, in contrast to the positive slope that was expected. These CAPM problems led Fama and French (1993) to introduce their three-factor model (3F) that exhibited a greater explanatory power over the CAPM, but displayed significant alphas suggesting that a portion of the returns remained unexplained. Carhart (1997) added the momentum factor to the 3F model and found improvements in explanatory power with his four-factor model (4F) relative to the CAPM and 3F for US stock mutual funds. Later on, Keene and Peterson (2007) added an illiquidity factor to the 4F model and found evidence of the superiority of this five-factor model (5F) in the US stock market.

The purpose of this study is to investigate if these multifactor models offer different and reasonable, in the practical sense, estimates of the cost of equity capital for Brazilian listed companies. Reasonable figures for the cost of equity capital would be those that analysts could consider in real life valuations. For example, figures below the risk-free rate are not reasonable as well as those that are too high relative to those of the market or industry peers. This study, thus, aims to answer if the additional complexity that comes with multifactor models generates estimates of the cost of equity capital that are significantly different from those obtained with the CAPM and with significantly greater explanatory power of the multifactor models. The importance to use adequate cost of equity capital estimates in valuation and corporate decision-making is a justification for this study. Inadequate cost of capital estimates may lead to under-investment due to the rejection of good projects when the cost of capital is over-estimated. On the other hand, if the cost of capital is under-estimated, one may overpay for an asset or accept projects that are too risky or not as good as one hoped.

In a related article, Estrada (2011) estimated the cost of equity capital using the CAPM and 3F models for the 30 companies that make up the Dow Jones Index from 2005 to 2009 and found little difference between the estimates obtained using each model (average difference of -0.2%). Argolo, Leal and Almeida (2012) estimated the out-of-the-sample cost of equity capital of 9 portfolios of Brazilian stocks using the CAPM and the 3F models from 1995 to 2007 and found that estimates obtained with the Fama and French (1993) model could be considered too high to be of practical use, casting doubt on the applicability of the three-factor model in the Brazilian market. Studies have also been carried out to verify the validity of multifactor models to describe the historical returns of Brazilian stocks. Malaga and Securato (2004), Santos, Famá and Mussa (2012), Rayes, Araújo and Barbedo (2012) and Machado and Medeiros (2011) are some examples and, in general, they show improvements in the explanatory power of multifactor models over the CAPM and alphas similar to the international literature, even though Rayes, Araújo and Barbedo (2012) suggest that the 3F model no longer contributes to explain the past returns of Brazilian stocks.

This study contributes to this domestic literature because it examines estimates for the cost of equity capital of the largest and most liquid listed companies in Brazil, instead of using portfolios of stocks, uses the 4F and 5F models, in addition to the CAPM and 3F models in the Malaga and Securato

(2004) and Argolo, Leal and Almeida (2012) studies, uses a more recent period in which Rayes, Araújo and Barbedo (2012) cast doubt about the contribution of the 3F model, and follows a practical approach in examining estimates of the cost of equity capital for each Brazilian company in the IBrX 100 index, following Estrada (2011) and Argolo, Leal and Almeida (2012).

To obtain the coefficients for each model, the return of the IBrX 100 index stocks in excess of the risk-free rate were regressed on risk factors. The risk factors were obtained from the Brazilian Center of Research in Finance of the Department of Economics of the University of São Paulo (NEFIN/USP). These risk factors can be freely downloaded from the NEFIN/USP website and allow for the quick and easy implementation of the 3F, 4F, and 5F multifactor models. Given the applied nature of this investigation, other multifactor models whose risk factors are not easily obtained, were not the considered in this study. The coefficients obtained for each risk factor of each model were multiplied by estimates of the expected value of the corresponding risk factor to obtain the cost of equity capital estimates for each company. The study then analyses the distribution of these estimates and the differences among them as well as the marginal contribution of each additional risk factor for each company.

The results show that the means of the cost of equity capital obtained with the four-factor and five-factor model are different from the CAPM estimates for the full sample period. The CAPM beta is positive and significant for the vast majority of stocks whereas most coefficients of the additional factors in the multifactor models are not positive and significant. The Wald test results suggest that the extra risk factors provide little contribution to the explanation of returns and, in most cases, should not be included in the models. The estimates obtained with the CAPM seem to be at least as good as those obtained using multifactor models with the added bonus of simplicity, which is an important practical implication of this study.

Literature Review

The Development of Multifactor Models

The cost of equity capital remains a central point of discussion in the finance literature due to its importance in valuation. The traditional form of the CAPM has been widely used in organizations as well as taught in business schools (Estrada, 2011; Fama and French, 2004). Graham and Harvey (2001) conducted a survey with 392 Chief Financial Officers (CFOs) in the US and Canada and found that 73.5% of them use the CAPM. Welch (2008) found that 75% of finance professors recommend the use of the CAPM to compute the cost of equity capital. Bancel and Mittoo (2014) surveyed 365 valuation experts across 10 European countries that hold a Chartered Financial Analyst (CFA) or similar designation and found that nearly 80% of them use the CAPM. Campos, Jucá and Nakamura (2016) surveyed 40 Brazilian public companies and assert that they use the CAPM to estimate the cost of equity capital. The popularity of the CAPM can be attributed to its simplicity (Fama and French, 2004). The main idea behind the model is that the return of a given security can be explained as a function of market risk. Based on strong assumptions, the model has received many criticisms, such as the one from Roll (1977), who argued that the market portfolio cannot be obtained, hence the CAPM cannot be tested.

In the 1980s, evidence began to emerge suggesting that new factors could play a role on the explanation of average returns. Fama and French (2004) report on some return anomalies or regularities that other authors identified, highlighting the small capitalization stocks effect, revealed through the persistent larger returns of small companies, and the value stocks effect, suggested by the better performance of high book-to-market (BE/ME) ratio stocks relative to those with low BE/ME. Considering that these regularities should be priced, Fama and French (1992) proceeded to assess the ones that contributed to explain the cross-section of average expected US stock returns. They indicated that BE/ME was a better representative of the value effect than the earnings-price ratio (E/P) and even of the leverage effect. The Fama and French (1993) three-factor model, thus, resulted from this line of research and included factors based on the size and value effects as additional risk factors to the CAPM's market risk premium (MRP).

The risk factors in addition to the MRP consisted of portfolios that mimic the size effect, yielding a small stock risk premium, and the value stock effect, yielding a value stock risk premium, measured using the BE/ME ratio. In order to test their model, Fama and French (1993) used the time-series regression methodology documented in Black et al. (1972) where stocks are grouped into portfolios and their excess returns were regressed on risk factors. The MRP was obtained by taking the value-weighted monthly return of a portfolio of stocks listed on the three largest US stock exchanges of the time in excess of the US Treasury bill. The details about the building of the small minus big (SMB) and high minus low

(HML) risk factors are discussed in detail in Fama and French (1993). SMB is based on the market value size of the company whilst HML is based on the BE/ME ration.

The size and value anomalies were naturally not the only regularities identified in the literature. Fama and French (2004) reported that the momentum effect was the most serious challenge faced by the 3F model. The momentum effect was documented in Chan, Jegadeesh and Lakonishok (1996) by means of a strategy that yields a positive return by holding stocks that performed well in the last 12 months and shorting those that performed poorly during the same period. Fama and French (1996) asserted the three-factor model could not explain the momentum effect. Lui, Strong and Xu (1999) showed that past winner stocks offered future abnormal returns in the UK as well and that the three-factor model did not account for this effect. Carhart (1997) then added the winner minus loser risk factor (WML) to the Fama and French (1993) model to obtain a four-factor model.

Amihud (2002) reported on another anomaly related to market illiquidity identified in his previous works. He argued that expected market illiquidity displayed a positive and significant impact on ex ante excess stock returns. Keene and Peterson (2007) added the illiquid minus liquid (IML) risk factor to the Carhart (1997) four-factor model. They found that IML explained a portion of the shared variation in US stock returns in the period from 1963 to 2002. This 5-factor model was used in Keene and Peterson (2007) and in this study as well.

Other multifactor models have been proposed in the literature. They will not be the focus of this study because their risk factors are not readily available for Brazilian stocks yet and, thus, given the applied nature of this investigation, this renders it more difficult for practitioners to use them to estimate the cost of equity capital. One of these models is the Fama and French (2015) five-factor model that adds the investment (Conservative minus Aggressive - CMA) and the profitability (Robust minus Weak - RMW) factors to their 3F model. The authors tested this new model from 1963 to 2013 in the US market and found that it explained between 71% to 94% of the cross-section variation of expected returns of the portfolios examined.

Multifactor models in Brazil

The earlier Brazilian evidence about the size and value anomalies was not always consistent with the US evidence. Costa Jr. and Neves (2000) investigated which variables better contributed to explain the cross-section of average returns in a study comprising 117 stocks listed in the São Paulo stock exchange from 1986 to 1996. They found that the most significant risk factors were the market risk premium and the BE/ME ratio, similarly to the Fama and French (1992) results. Rodrigues and Leal (2003) investigated the Fama and French (1993) model in the period of 1991 to 2001 and found evidence supporting the value effect in the Brazilian market, however, the sign of the coefficients of the size premium was negative suggesting a large company effect that was contrary to the international literature. Malaga and Securato (2004) analyzed the period of from 1995 to 2003 with a sample containing all non-financial listed stocks that had 12 consecutive months of stock prices and non-negative shareholder equity at the end of the preceding year. They found that the 3F model offered superior explanatory power on the variation of returns compared to the CAPM. The coefficient of the size premium (SMB) was negative, in line with previous Brazilian findings and contrary to Fama and French (1993).

Argolo, Leal and Almeida (2012) also tested the Fama and French (1993) model in the Brazilian market from 1995 to 2007 and compared it to the CAPM, but their goal was to ascertain whether the three-factor model could provide reasonable out-of-the-sample estimates of the cost of equity capital. They built nine portfolios of stocks in a way similar to the Fama and French (1993) approach and ran ordinary least squares (OLS) regressions to estimate the coefficients for the CAPM and the 3F model. They found that the CAPM beta was significant at a 5% level in all portfolios. Moreover, beta remained significant in the three-factor model and 5 out of 9 portfolios had the three coefficients significant at a 5% level. The adjusted R^2 increased with the addition of the risk factors, similar to previous studies (Málaga and Securato, 2004; Fama and French 1993).

Subsequently, these authors estimated the cost of equity capital for each portfolio using historical averages as estimates for the expected value of each risk factor and multiplied these expected risk factor estimates by their corresponding regression coefficients. They concluded that these cost of equity capital estimates were too high for the three-factor model to be useful estimate in practical cost of capital applications.

As examples of more recent tests of multifactor models in Brazil, Rayes, Araújo and Barbedo (2012) tested the Fama and French (1993) model for 40 stocks from 2000 to 2008. They grouped stocks into equally weighted and value weighted portfolios. In addition, they tested the Fama and French (1993) model with individual stocks. They found that the beta coefficient is significant for most regressions but the Fama and French (1993) factors no longer explain the variation of returns for these Brazilian stocks, in contrast to previous articles.

Mussa, Famá and Santos (2012) investigated the Carhart (1997) model in Brazil from 1995 through 2006. They compared their findings to the CAPM and the three-factor model of Fama and French (1993). The authors concluded that the 4F model offered greater explanatory power relative to the 3F model and that the 3F model showed greater explanatory power relative to the CAPM. The authors also highlighted the fact that the coefficient of SMB was negative, contrary to international literature but corroborating previous Brazilian studies such as Málaga and Securato (2004) and Rodrigues and Leal (2003).

Regarding the illiquidity factor, Vieira and Milach (2008) investigated a series of proxies for liquidity and found that the bid-ask spread as well as the illiquidity factor had a positive relationship with expected returns in a study for the 1995-2005 period. Machado and Medeiros (2011) also analyzed whether an illiquidity premium existed in the Brazilian stock market and concluded that it contributed to the explanation of expected returns when included in a multifactor asset pricing model. Carvalho et al. (2021) also investigated if the liquidity risk influences the performance of the Fama and French's (2015) five-factor model. They analyzed 385 shares during the period from June 1999 to June 2017 and showed that this risk factor improves the model.

Machado, Faff and Silva (2017) investigated the variations in Brazilian stock returns during the period from June 1997 to June 2014. They used time series and cross-section regressions, according to the Fama and French's (2015) five-factor model. They found that only three factors are relevant to stock returns: B/M, momentum and liquidity. Furthermore, they did not find significance for the additional investment and profitability factors.

Finally, Moreira et al. (2021) observed that structural breaks caused by economic instability can affect the relationships verified, especially the size and level of investment. They analyzed the companies present in B3 in the period between 2006 and 2017 and selected sub-periods according to the economic situation (pre-international crisis, international crisis, post-international crisis and national recession). Thus, it can be understood that according to the results presented, the risk factors in the five-factor Fama and French model, present additional behavior to the economic situation of the country and that this adaptation of the behavior is necessary for the development of the capital market as a whole.

Methodology

Sample

The sample consists of the stocks that made up the IBrX 100 in October of 2018, which was the time of data collection. The composition of the IBrX 100 changes periodically due to the eligibility criteria used to select the stocks that comprise it, however, in this study, no changes were made in the sample, keeping the same 100 stocks that comprised the index in October of 2018 over the full period. The IBrX 100 portfolio was chosen because it has more stocks than the Ibovespa, yielding a larger sample. The index contains the 100 most representative and traded stocks in Brasil Bolsa Balcão (B3), as the consolidated Brazilian exchange is currently known, because it considers their market value and liquidity in the inclusion criteria. The sample period goes from July 2008 to June 2018 in order to obtain a 10-year sample ending at the time of data collection. All stocks with less than 36 consecutive monthly observations were deleted from the sample, resulting in 91 stocks for the full sample period. The 10-year period was chosen because a longer period would result in a bigger number of stocks being deleted from the sample due to the exclusion criterion. A table with the stocks deleted from the sample and their number of observations can be obtained with the authors but was omitted to save space. No exclusions were made regarding the simultaneous presence of common or preferred stocks, so a company could have both kinds of stocks considered in the analysis. One should note that preferred stocks in Brazil are similar to common stocks but usually offer no voting rights. They do not commonly offer fixed dividends. The IBrX 100 is weighted by the market value of the free floating shares while the Ibovespa was weighed by market liquidity up to December of 2013 and only after this point it began to be weighted by the market value of free floating shares (Roquete, Leal & Campani, 2018). In spite of their differences, these indices have a correlation of 98,7% in the sample period.

The dividend and split adjusted monthly closing stock prices and the Interfinancial Certificate of Deposit (CDI) rate monthly yield were obtained from the Economatica database. The CDI was chosen to compute the excess returns of stocks because the CDI is the standard benchmark of the opportunity cost of investors and, as such, it is commonly reported in the factsheets of mutual funds in the country. It displayed a 99% correlation with the short-term Brazilian Treasury bill rate (Roquete, Leal & Campani, 2018). The monthly stock returns were calculated using Equation 1, where $R_{s,t}$ is the month t return of stock s, $P_{s,t}$ is the closing price of the last trading day of month t of stock s and $P_{s,t-1}$ is the closing price of the last trading day of month $t-1$ of stock s.

$$R_{s,t} = \frac{P_{s,t}}{P_{s,t-1}} - 1 \qquad (1)$$

The monthly returns of the MRP, SMB, HML, WML, and IML risk factors were obtained from NEFIN/USP and the procedures described below are those detailed in NEFIN (2015). The market portfolio is the one used by NEFIN/USP and is made up of the most traded stock of a firm (the one with the highest trading volume of year *t-1*). In addition, a market portfolio stock has to be traded in more than 80% of the days in year *t-1* with volume greater than R$ 500.000,00 per day and must be listed prior to December of year *t-1*. NEFIN/USP uses the daily equivalent of the 30-day DI Swap rate as the risk-free rate. This is a fixed for floating swap rate. NEFIN/USP computes the difference between the value-weighted monthly return of their market portfolio and the monthly return of their risk-free rate to obtain the MRP. The MRP computed using the IBrX 100 and the Ibovespa is the monthly return of the index minus the CDI monthly yield. These monthly risk premia have a correlation of 99.5% and 98.2%, respectively, with the MRP computed by NEFIN/USP, which is the MRP representation in this study.

SMB is the return of a portfolio long on stocks with low market capitalization (small) and short on stocks with high market capitalization (big). Every January of year *t*, the eligible stocks are sorted in ascending order according to their December of year *t-1* market capitalization. The stocks are then separated into 3 terciles. The equally-weighted returns of the first portfolio (small stocks) and the third portfolio (big stocks) are then computed. The SMB factor is the return of the small stocks portfolio minus the return of the big stocks portfolio (NEFIN, 2015).

HML is the return of a portfolio long on stocks with high BE/ME ratios and short on stocks with low BE/ME ratios. Every January of year *t*, the eligible stocks are sorted in ascending order into 3 terciles (portfolios) according to the BE/ME ratio of the stocks in June of year *t-1*. Then, the equally-weighted returns of the first portfolio (low BE/ME) and the third portfolio (high BE/ME) are computed. The HML factor is the return of the high BE/ME stock portfolio minus the return of the low BE/ME stock portfolio (NEFIN, 2015).

WML is the return of a portfolio long on stocks with high past returns and short on stocks with low past returns. Every month *t*, the eligible stocks are sorted in ascending order and divided into 3 terciles (portfolios) according to their cumulative returns between month *t-12* and *t-2*. Then, the equally-weighted returns of the first portfolio (low past returns, i.e., "losers") and the third portfolio (high past returns, i.e.," winners") are computed. The WML factor is the return of the winners' portfolio minus the return of the losers' portfolio (NEFIN, 2015).

Finally, IML is the return of a portfolio long on illiquid stocks (high illiquidity) and short on liquid stocks (low illiquidity). Every month *t*, the eligible stocks are sorted in ascending order and divided into 3 terciles (portfolios) according to their previous twelve-month moving average of illiquidity, as in Amihud (2002). Then, the equal-weighted returns of the first (high illiquidity) and the third portfolios (low illiquidity) are computed. The IML factor is the return of the illiquid stocks portfolio minus the return of the liquid stocks portfolio (NEFIN, 2015).

Regression models

Time-series regressions with robust standard errors were used to estimate the coefficients for each model. The robust standard errors provide a way to obtain consistent (in the statistical sense) estimates of the variances and covariances of OLS estimators even if there is heteroscedasticity (Gujarati, 2004, p. 417). According to Fama and French (1993), the benefits of using time-series regressions are twofold: (1) if assets are priced rationally, variables such as size and BE/ME that are related to average returns, must proxy for sensitivity to common risk factors in returns; and (2) excess returns are used in the dependent variable as in the independent variables and, therefore, the regression should produce intercepts that are indistinguishable from 0. The expressions for the CAPM, three-factor, four-factor and five-factor models are in Equations 2 to 5, respectively, where $R_{s,t} - Rf_t$ is the monthly return of stock *s* in excess of the risk free rate (CDI), MRP, SMB, HML, WML, and IML risk factors were described above, and ε_t is the residual of the model. The correlation among the MRPs computed using CDI and the DI swap are very close to 1, thus, the DI swap and the CDI could be used interchangeably.

$$R_{s,t} - Rf_t = \alpha + \beta(MRP_t) + \varepsilon_t \qquad (2)$$

$$R_{s,t} - Rf_t = \alpha + \beta(MRP_t) + s(SMB_t) + h(HML_t) + \varepsilon_t \qquad (3)$$

$$R_{s,t} - Rf_t = \alpha + \beta(MRP_t) + s(SMB_t) + h(HML_t) + w(WML_t) + \varepsilon_t \qquad (4)$$

$$R_{s,t} - Rf_t = \alpha + \beta(MRP_t) + s(SMB_t) + h(HML_t) + w(WML_t) + i(IML_t) + \varepsilon_t \qquad (5)$$

Cost of equity capital computation

The estimates of each risk factor used in this study to estimate the expected cost of equity capital are the arithmetic average of the monthly returns for the full period of data available in the NEFIN/USP website, which was from January of 2001 to June of 2018 at the time of data collection. The reason for choosing the longest period available is due to the standard error being smaller, similarly to Estrada (2011), who estimated the cost of equity capital for individual stocks using the Fama and French (1993) risk factors averaged from 1927-2009. The use of the arithmetic average, as pointed out in Ibbotson Associates (2007), can be shown to be more suitable when dealing with expected rate of return, as opposed to geometric averages which are better for reporting past returns. After obtaining the set of coefficients for each individual stock with each model, the cost of equity capital was estimated for every stock through the multiplication of each risk factor average in Table 1 by the corresponding estimated risk factor coefficient. After multiplying each risk factor average by its respective estimated coefficient, the full-period (2001-2018) risk-free (CDI) average is added to estimate the cost of equity capital. The result is then annualized. Equation 6 describes the procedure for the single-factor model. R_e is the expected rate of return of any given stock and Rf_t is the risk free rate (CDI).

The risk factor portfolios obtained from NEFIN/USP are provided as daily returns. To compute the monthly averages in Table 1 the cumulative returns are necessary. The cumulative returns are obtained by adding 1 to the daily return of the risk factor and then multiplying the result of day *t* by the result of the previous day. After the cumulative returns were computed, the accumulated return on the last trading day of month *t* is divided by the accumulated return of the last trading day of the month *t-1* and subtracted by 1. This procedure is similar to the one used to compute the monthly return of stocks in this study.

$$R_e = \left(1 + Rf_t + \beta(MRP_t)\right)^{12} - 1 \qquad (6)$$

Table 1.
Monthly arithmetic average of risk factors and risk free rate

	Risk-Free	MRP	SMB	HML	WML	IML
2001-2018	0.0103	0.0022	-0.0004	0.0055	0.0120	0.0002
2008-2018	-	-0.0027	-0.0033	-0.0007	0.0122	-0.0034
Correlation - Sample period (2008-2018)						
	MRP	**SMB**	**HML**	**WML**	**IML**	
MRP	1.00					
SMB	0.42	1.00				
HML	0.30	0.31	1.00			
WML	-0.46	-0.61	-0.31			
IML	0.11	0.80	0.34	-0.31	1.00	

Note. The full sample period goes from July 2008 to June 2018. The full time series of the risk factors in the NEFIN/USP database went from January 2001 through June 2018.

In order to compare the asset-pricing models, the adjusted R^2 will be analyzed. A Wald test will be performed to assess the contribution of each risk factor to the models. As a robustness test, a sub-period analysis will be carried out. The full sample (July 2008 through June 2018) contains 120 observations. The more recent sub-period (July 2013 through June 2018) contains 60 observations as does the earlier sub-period (July 2008 through June 2013). The same methodology used for estimating the cost of equity capital for the full period was used for the sub-periods. The more recent sub-period contained 91 stocks and the earlier 84 stocks. This is due to the previously mentioned criterion of deleting any stocks from the sample that had less than 36 monthly observations as well as due to the sample selection consisting of the IBrX 100 portfolio in October 2018. Naturally, as one goes towards past periods, some companies listed in October 2018 were not listed yet.

Results

A table containing the estimated cost of equity capital for individual stocks for the full sample period from July 2008 to June 2018, as well as for each sub-period, using the CAPM and the multifactor models is too large but is available with the authors. Table 2 contains a summary of the estimates and of the differences between model estimates with descriptive statistics and the p-value of the Shapiro-Wilk normality test. The average difference between CAPM estimated cost of equity capital and the other models is very low. This is consistent with the conclusions of Argolo, Leal and Almeida (2012) for portfolios of stocks that claim that using the CAPM in Brazil would be best and with the practice of Brazilian managers reported in Campos, Jucá and Nakamura (2016), but contrasts with Málaga and Securato (2004). Normality is rejected for the 3F cost of capital distribution and for three difference distributions at the 5% level. The differences distributions show a high dispersion. It is also interesting to note that the CAPM estimates of the cost of equity capital shows the smallest range and standard deviation. The additional factors seem to increase the variability of the distribution of the cost of equity capital. However, the addition of IML relative to the 4F model does not seem to change the distribution of the cost of equity capital at all, as it displays very a small range and standard deviation. This result of the liquidity risk factor contrasts with the results found by Machado and Medeiros (2011) and Carvalho et al. (2021) who claim that the liquidity factor would add explanatory power for some stocks, at least.

Table 2.
Descriptive statistics of the cost of equity capital

Variable	Mean	T-stat.	Median	Z-stat.	Std. Dev.	Max	Min	SW	N
CAPM	0.159**	107.5	0.158**	8.3	0.014	0.204	0.129	0.073	91
3F	0.155**	33.6	0.145**	8.3	0.044	0.318	0.052	0.000	91
4F	0.146**	27.2	0.142**	8.3	0.051	0.288	0.010	0.553	91
5F	0.147**	29.1	0.143**	8.3	0.048	0.274	-0.035	0.053	91
3F-CAPM	-0.004	-1.1	-0.008	-1.4	0.038	0.114	-0.106	0.676	91
4F-CAPM	-0.013**	-2.3	-0.008	-1.8	0.055	0.100	-0.157	0.124	91
4F-3F	-0.009	-1.6	0.002	-0.4	0.052	0.081	-0.218	0.000	91
5F-CAPM	-0.012**	-2.3	-0.014	-1.8	0.051	0.086	-0.195	0.026	91
5F-3F	-0.008	-1.5	0.000	-0.5	0.048	0.076	-0.199	0.000	91
5F-4F	0.001	0.7	0.000	0.4	0.014	0.038	-0.046	0.060	91

Note. Null hypothesis for the t-test is that the mean of the distribution is equal to zero. Null hypothesis of the Wilcoxon signed-rank test is that the median is equal to zero (z-stat). "SW" is the Shapiro-Wilk normality test and its null hypothesis is that the population is normally distributed. 3F is the Fama and French (1993) three-factor model. 4F is the Carhart (1997) four-factor model. 5F is the five-factor model of Keene and Peterson (2007). 3F-CAPM, 4F-CAPM, 4F-3F, 5F-CAPM, 5F-3F and 5F-4F are the average differences between the estimated cost of equity capital between these pairs of models. ** and * indicate significance at the 1% and 5% levels, respectively.

Table 2 also shows the results for the one-sample t-test of the means and the Wilcoxon signed rank test of the medians for the estimates. The t-test assumes that the population is normally distributed and the Wilcoxon test is a non-parametric hypothesis test. The null hypotheses of the t-test of the means are rejected for the difference distributions 4F-CAPM and 5F-CAPM, suggesting that the means are different, however, for the remaining differences distributions, the null hypotheses could not be rejected, suggesting their means and medians are the same and null across models. The results are consistent with those authors that claimed that the factors added to the traditional CAPM formulation could offer a different cost of capital estimate, such as Estrada (2011) and Málaga and Securato (2004).

Table 3.

Average regression diagnostics

Model	Adj R^2	F	Ramsey Test (p-value)	VIF	BP (Chi^2)	BG (Chi^2)	N
CAPM	28.7%	46.79	0.30 (25)	1.00 (0)	3.54 (21)	0.00 (0)	91
3F	33.2%	22.37	0.25 (30)	1.28 (0)	3.82 (19)	0.00 (0)	91
4F	33.9%	19.53	0.24 (28)	1.52 (0)	3.78 (20)	0.00 (0)	91
5F	34.1%	16.39	0.24 (30)	2.90 (0)	3.67 (23)	0.00 (0)	91

Note. The numbers in parenthesis are the count of regressions that exhibit a diagnostic problem. The Ramsey Test tests for omitted variables. VIF is the mean of the Variance Inflation Factor and tests for multicollinearity with a threshold of 10. BP is the Breusch-Pagan test for heteroscedasticity. BG is the Breusch-Godfrey test for serial correlation. 3F is the Fama and French (1993) three-factor model. 4F is the Carhart (1997) four-factor model. 5F is the five-factor model of Keene and Peterson (2007). All values are averages except for N.

Table 3 shows the averages of selected diagnostics of the regressions. The individual results for each regression can be obtained with the authors. The Ramsey Regression Equation Specification Error Test (RESET) is used to verify if there is a general functional form misspecification. The null hypothesis for the RESET test is that the model is correctly specified. The average p-value for the diagnostics suggests that the null hypothesis cannot be rejected for more than two-thirds of the models. The variance inflation factor (VIF) quantifies by how much the variance of each estimated coefficient is inflated. The standard errors, and therefore the variances, are considered inflated when there is multicollinearity. Setting a threshold value for VIF in order to conclude that multicollinearity exists is arbitrary. Often times, the value 10 is chosen to decide that multicollinearity is a problem (Wooldridge, 2008, p. 99). The CAPM VIF is naturally 1 since it is a single-factor model, as more factors are added, the VIF tends to increase due to a possible correlation among the dependent variables, nevertheless, the average variance inflation factor is not high enough to indicate that there is any multicollinearity problem in the models. The null hypothesis of the Breusch-Pagan (BP) test is that the error variances are all equal versus the alternative hypothesis that the error variances are different. In essence, the BP test measures how errors increase across the explanatory variable. The average chi-square value of the test in Table 3 suggests that there is no heteroscedasticity in more than two-thirds of the models. Finally, the Breusch-Godfrey (BG) test verifies the presence of serial correlation in the model. The null hypothesis states that there is no serial correlation. The average in Table 3 suggests that there is no serial correlation in the residuals in all models.

Showing all individual coefficients for each model would be impractical, thus, Table 4 summarizes them and shows their mean and significance levels as well as the number of positive and significant coefficients in parenthesis. Even though the average alpha, beta and ‘s’ coefficients in Table 4 are significantly different from zero for the four models, there are very few significant alphas in the individual models. This apparent contradiction is due to their standard deviation being close to their average, producing a high t-statistics. The average beta is close but below 1 and nearly all of them are positive and significant in the individual regressions, consistently with previous Brazilian studies such as Rayes, Araújo and Barbedo (2012) and Machado and Medeiros (2011). The averages of "s" are positive and nearly 30% of them are positive and significant in the individual models. Thus, in most cases "s" is not positive or significant, as would be expected from Fama and French (1993), consistently with previous Brazilian studies such as Rayes, Araújo and Barbedo (2012) and Malaga and Securato (2004). It is also interesting to note that the presence of the IML risk factor has reduced the number of positive and significant "s" coefficients, this is probably due to the high correlation of 80% between SMB and IML portrayed in Table 1. The averages of the ‘h’, 'w', and 'i' coefficients are not statistically significant for the four models tested and very few models presented "w" and "i" significant. The small number of positive and significant "h" coefficients is consistent with the more recent evidence in Rayes, Araújo and Barbedo (2012). However, the very few positive and significant "w" and "i" coefficients for individual stocks contrasts with the results of Santos, Famá and Mussa (2012) and Machado and Medeiros (2011) for portfolios of stocks. For the three-factor model, only 2 regressions have all of their coefficients positive and significant at the same time and for the 4F and 5F models, no regression showed these results.

Table 4.
Average of Model Coefficients

Model	Alpha	Beta	s	h	W	i	Significant	N
CAPM	0.01^{**} (5)	0.97^{**} (87)	-	-	-	-	87	91
3F	0.01^{**} (8)	0.89^{**} (87)	0.28^{**} (27)	-0.02 (17)	-	-	2	91
4F	0.01^{**} (6)	0.88^{**} (87)	0.25^{**} (26)	-0.02 (17)	-0.05 (6)	-	0	91
5F	0.01^{**} (8)	0.87^{**} (85)	0.28^{**} (14)	-0.01 (16)	-0.04 (5)	-0.04 (4)	0	91

Note. The number of positive and significant coefficients are in parenthesis. Alpha is the intercept. Beta is the coefficient of the MRP. "s" is the coefficient of SMB. "h" is the coefficient of HML. "w" is the coefficient of WML. "i" is the coefficient of IML. "Significant" is the number of regressions with all coefficients positive and significant. N is the number of stocks (models). ** and * indicate significant at the 1% and 5% levels, respectively, for the null hypothesis that the average coefficient is 0.

A Wald test assesses the contribution that each additional risk factor brings to the CAPM model. The results are in Table 5. The Wald test, as described in Agresti (1990, p. 12), is a way of testing whether the coefficients associated with one or a group of explanatory variables are zero. If the Wald test is significant for a specific explanatory variable or group of variables, then it is possible to conclude that the parameters associated with these variables are not zero and that they should be included in the model. If, on the other hand, the test is not significant, the variables should not be included in the model. An F-statistic equal or greater than 4 is usually significant at a 5% level, depending on the degrees of freedom.

Table 5.
Wald Test

Model	Null Hyp.	F	Null Rejection	N
3F_CAPM	s=0; h=0	4.392	39	91
4F_CAPM	s=0; h=0; w=0	3.756	36	91
5F_CAPM	i=0; w=0; s=0; h=0	3.273	28	91
4F_3F	w=0	2.139	16	91
5F_3F	i=0; w=0	1.860	13	91
5F_4F	i=0	1.651	9	91

Note. 3F_CAPM indicates the contribution of the two additional factors of 3F relative to the CAPM. The same logic follows in the other rows. F is the average of the individual F statistics. “Null Rejection” shows the number of coefficients that are significant at the 5% level for the Wald Test. N is the number of models (stocks).

The 3F_CAPM test tries to answer if the two additional factors in the three-factor model (SMB and HML) add to the explanation of average returns relative to the CAPM. In this case, the null hypothesis states that the coefficient 's' and 'h' are both equal to zero. In 39 out of 91 regressions, the F statistics was significant, and therefore, for these specific cases, the SMB and HML factors should be included in the model. The same rationale can be applied to each of the other rows in Table 5. The 5F_4F test shows that only in 9 cases out of 91 the IML factor should be added to the 4F model, casting doubt on the applicability of this five-factor model for any given stock. Adding the WML factor to the Fama and French model (4F_3F) does not seem to provide a large improvement since only in 16 cases the momentum risk factor adds to the explanation of returns. Also, when taking into account the results of Table 4, in which the 'w' coefficient was positive and significant in only 6 cases out of 91, the addition of the WML risk factor seems to be dismissible for estimating the cost of equity capital of individual stocks in this sample. The results for "h", "w", and "i" contrast with those presented in Machado and Medeiros (2011) and Santos, Famá and Mussa (2012) for portfolios of stocks. Thus, the multifactor models do not seem to generate significant coefficients for individual stocks.

Sub-Period Analysis

In order to verify stability across time, the sample was divided into two sub-periods: the most recent half (July 2013 through June 2018) and the earlier half (July 2008 through June 2013). Detailed tables with the estimated cost of equity capital for the sub-periods for each company and model are available with the authors. Table 6 shows the descriptive statistics for the estimated cost of equity capital and the difference distributions as well as the p-value of the Shapiro-Wilk normality test for each sub-period**.** The normality of the cost of equity capital distributions is not rejected for various models and differences distributions. There is evidence to suggest that the means of the estimated cost of equity capital with each model, when compared among them, are the same at the 5% level, except for 4F and 5F relative to the CAPM in the more recent. The numbers also suggest that the medians of the 3F, 4F and 5F are different relative to the CAPM at the 5% level. The results in Panel A are similar to those for the full period, with the exception of the results of the Wilcoxon test. Panel B results suggest that the means and medians of the cost of equity capital of the four models are the same, similarly to the full period with the exception of the means of 4F and 5F relative to the CAPM. The behavior of the dispersion of the distributions in the sub-periods is similar to what was verified in the full period.

Table 6.
Descriptive statistics of the cost of equity capital per sub-period

Variable	Mean	T-stat.	Median	Z-stat.	Std. Dev.	Max	Min	SW	N
Panel A: 2013-2018									
CAPM	0.161**	80.5	0.160**	8.3	0.019	0.232	0.117	0.001	91
3F	0.153**	27.4	0.146**	8.3	0.053	0.366	0.052	0.000	91
4F	0.145**	25.0	0.152**	8.3	0.055	0.280	0.010	0.131	91
5F	0.142**	25.6	0.143**	8.3	0.053	0.270	-0.035	0.316	91
3F-CAPM	-0.008	-1.8	-0.001**	-3.0	0.044	0.139	-0.106	0.000	91
4F-CAPM	-0.016**	-2.5	-0.008**	-2.2	0.061	0.134	-0.176	0.060	91
4F-3F	-0.008	-1.1	0.002	0.3	0.069	0.126	-0.315	0.000	91
5F-CAPM	-0.019**	-3.2	-0.015**	-2.9	0.057	0.124	-0.194	0.654	91
5F-3F	-0.011	-1.7	-0.005	-1.2	0.060	0.115	-0.273	0.000	91
5F-4F	-0.003	-1.4	-0.006	-1.8	0.021	0.046	-0.050	0.079	91
Panel B: 2008-2013									
CAPM	0.158**	94.8	0.159**	8.0	0.015	0.192	0.131	0.081	84
3F	0.159**	39.4	0.159**	8.0	0.037	0.296	0.081	0.014	84
4F	0.152**	20.9	0.159**	8.0	0.067	0.300	0.012	0.337	84
5F	0.153**	21.4	0.159**	8.0	0.066	0.301	0.025	0.243	84
3F-CAPM	0.002	0.4	-0.004	-0.1	0.038	0.120	-0.077	0.040	84
4F-CAPM	-0.006	-0.7	0.005	-0.3	0.074	0.138	-0.180	0.064	84
4F-3F	-0.007	-1.1	-0.001	-0.9	0.060	0.151	-0.141	0.424	84
5F-CAPM	-0.005	-0.6	0.003	-0.2	0.073	0.139	-0.166	0.059	84
5F-3F	-0.006	-1.0	0.000	-0.8	0.059	0.152	-0.131	0.474	84
5F-4F	0.001	1.7	0.000	1.3	0.004	0.013	-0.010	0.069	84

Note. Null hypothesis for the t-test is that the mean of the distribution is equal to zero. Null hypothesis of the Wilcoxon signed-rank test is that the median is equal to zero (z-stat). "SW" is the Shapiro-Wilk normality test and its null hypothesis is that the population is normally distributed. 3F is the Fama and French (1993) three-factor model. 4F is the Carhart (1997) four-factor model. 5F is the five-factor model of Keene and Peterson (2007). 3F-CAPM, 4F-CAPM, 4F-3F, 5F-CAPM, 5F-3F and 5F-4F are the average differences between the estimated cost of equity capital between these pairs of models. ** and * indicate significant at the 1% and 5% levels, respectively.

The analysis of the diagnostics of the regressions and of the average coefficients for each sub-period is qualitatively similar to those in Tables 4 and 5, respectively, and were omitted to save space. They are available with the authors. Table 7 shows the Wald test for the two sub-periods. The SMB and HML factors add to the explanation of returns and, therefore, improve the CAPM performance in 26% of the cases for the more recent sub-period and 33% of the cases for the earlier sub-period. The same rationale can be applied to the rest of the tests. There is little gain in adding the fifth risk factor (IML) to the Carhart

(1997) model in both sub-periods. Similarly, the risk factors WML and IML do not improve significantly on the three-factor model. In general, the Wald test shows that the extra risk factors added to the CAPM fail to provide significant explanatory power for the model for most stocks. The SMB and HML factors should not be included in the model to estimate the cost of equity capital for individual stocks in 67 cases out of 91 for the recent sub-period and in 56 out of 84 for the earlier sub-period. The number of regressions that should not bear extra risk factors is alarmingly large. The results for the full period show superior performance when assessing the contribution of extra risk factors to the CAPM compared to the sub-periods. However, in more than half of the regressions of the full period, the additional risk factors should not be included in the CAPM.

Table 7.
Wald test - Sub-periods B and C

			Sub-period B			**Sub-period C**	
Model	**Null Hyp.**	**F**	**Null Rejection**	**N**	**F**	**Null Rejection**	**N**
3F_CAPM	s=0; h=0	3.3	24	91	3.0	28	84
4F_CAPM	s=0; h=0; w=0	3.0	21	91	3.0	27	84
5F_CAPM	i=0; w=0; s=0; h=0	2.9	18	91	2.8	19	84
4F_3F	w=0	2.3	13	91	2.3	16	84
5F_3F	i=0; w=0	2.2	11	91	2.1	19	84
5F_4F	i=0	1.9	14	91	1.7	11	84

Note. 3F_CAPM indicates the contribution of the two additional factors of 3F relative to the CAPM. The same logic follows in the other rows. F is the average of the individual F statistics. “Null Rejection” shows the number of coefficients that are significant at the 5% level for the Wald Test. N is the number of models (stocks).

Conclusion

The purpose of this study was to understand if multifactor asset-pricing models offer different estimates of the cost of equity capital relative to the CAPM for individual stocks as well as to ascertain if the additional risk factors present an explanatory power gain. The results suggest that the means of the cost of equity capital estimated with the four-factor and five-factor models are different from the CAPM in the full sample period and also in the most recent sub-period. However, the evidence suggests that the medians are the same in the full period and in the oldest sub-period. Thus, there is no clear and conclusive evidence that the estimates of the cost of equity capital of the multifactor models are in general different from those produced by the CAPM.

In addition, most coefficients for multifactor regressions are not positive and significant. To decide which model to use based solely on the adjusted R^2 analysis would be misleading because the resulting estimate is built upon coefficients that are indistinguishable from zero in most cases. The Wald test was carried out in order to assess the contribution of each factor to the models with similar results for the full period and sub-periods. The additional risk factors should not be included in the models in the majority of cases. The coefficients of the SMB and HML risk factors were significant for the Fama and French (1993) model in 27 and 17 out of 91 regressions, respectively. Moreover, these coefficients are only positive and significant together in the same regression in only 2 cases. For the Carhart (1997) and the five-factor model, no regression showed all coefficients positive and significant at the same time. For most regressions, the market risk premium coefficient beta was positive and significant. Moreover, the average intercept was positive and significantly different from zero for the four models, even though very few of them were significant in the individual regressions.

This study raises concern about the applicability of multifactor models for individual stocks in the Brazilian market, which is consistent with the conclusions of Argolo, Leal and Almeida (2012) for portfolios of stocks and with the actual Brazilian practice reported in Campos, Jucá and Nakamura (2016). Thus, a practical implication is that estimates of the cost of equity capital obtained with the CAPM seem to be at least as useful, in most cases, as those obtained with multifactor models, which is important given the simplicity of the CAPM despite the many criticisms it has received over the years. Even though the multifactor models show higher adjusted R^2 when compared to the CAPM, the results imply that the complexity that comes with multifactor models in general outweigh the possible benefits.

Multifactor models have been extensively tested in the Brazilian market using portfolios of stocks (Rayes, Araújo & Barbedo, 2012; Santos, Famá & Mussa, 2012; Machado and Medeiros, 2011). A limitation of this study is that the estimates obtained running time-series regressions for individual stocks may be jeopardized by higher variances compared to those of portfolios of stocks, even though estimates of the cost of equity capital are important for individual stocks. Additionally, the high volatility in 2008 and 2009, the financial crisis year and the following rebound year, may have affected the sample. An interesting approach would be to group stocks of similar characteristics, such as from the same industry or size group, into portfolios in order to minimize variance, and to verify if a multifactor model would consistently offer better estimates of the cost of equity capital for the Brazilian market industries in a period of more stability, even though the earlier results of Argolo, Leal and Almeida (2012) suggest that this would not happen. A final limitation of this study is that it relies on the historical averages of each risk factor as estimates of their expected values. Even though it may be easy to obtain alternative expected values for the MRP and the risk-free rate, alternative estimates for the other risk factors are not so economically intuitive. Future research could discuss the issue of which estimates are better for the expected values of the risk factors for a practical application setting.

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